JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

Common Fixed Point Theorems in Three Complete Intuitionistic Generalized Fuzzy Metric Spaces

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Abstract:. In this paper, we have proved common fixed point theorems for three Intutionistic Generalized Fuzzy Metric Spaces which improve the results of Jain, Sahu and Fisher in three metric spaces. This result generalizes well known fixed point theorems in complete metric spaces.

Keywords: Common fixed point theorem, Generalized fuzzy metric spaces, Complete.
AMS Mathematics Subject Classification: 47H10, 54A40.

1. Introduction

It is notable that the fuzzy metric spaces are a speculation of the metric spaces, in light of the theory of fuzzy sets. Kramosil and Michalek [8] presented a fuzzy metric space playing out the probabilistic metric spaces way to deal with the fuzzy settings. Further on, George and Veeramani [4] modify the idea of fuzzy metric space. Fisher [2], Aliouche and Fisher [1], Telci [13] demonstrated a few related fixed point theorem in compact metric spaces. As of late, Rao et.al [10] demonstrated some related fixed point theorem in successively compact fuzzy metric spaces. Related fixed point theorem on three metric spaces have been examined by R. K. Jain et al. [5]. In this paper, we have demonstrated basic common fixed point theorems on three complete intuitionistic generalized fuzzy metric spaces which improve the consequences of R. K. Jain et al. [5].

2. Preliminaries

Definition 2.1. Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is said to be an IGFMS, if X is an arbitrary nonempty set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t - conorm, $\mathcal{M}$ and $\mathcal{N}$ are fuzzy sets on $^3 \times (0, \infty)$ satisfying the

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

following conditions:

For every $\alpha, \beta, \gamma, \delta \in X$ and $t, s > 0$.

(i) $\mathcal{M}(\alpha, \beta, \gamma, t) * \mathcal{N}(\alpha, \beta, \gamma, t) \leq 1$,

(ii) $\mathcal{M}(\alpha, \beta, \gamma, t) > 0$,

(iii) $\mathcal{M}(\alpha, \beta, \gamma, t) = 1$ if and only if $\alpha = \beta = \gamma$,

(iv) $\mathcal{M}(\alpha, \beta, \gamma, t) = \mathcal{M}(p(\alpha, \beta, \gamma), t)$, where p is a permutation function,

(v) $\mathcal{M}(\alpha, \beta, \delta, t) * \mathcal{M}(\delta, \gamma, \gamma, s) \leq \mathcal{M}(\alpha, \beta, \gamma, t + s)$,

(vi) $\mathcal{M}(\alpha, \beta, \gamma, .) : (0, \infty) \to [0, 1]$ is continuous,

(vii) $\mathcal{N}(\alpha, \beta, \gamma, t) > 0$.

(viii) $\mathcal{N}(\alpha, \beta, \gamma, t) = 0$ if and only if $\alpha = \beta = \gamma$,

(ix) $\mathcal{N}(\alpha, \beta, \gamma, t) = \mathcal{N}(p(\alpha, \beta, \gamma), t)$, where p is a permutation function,

(x) $\mathcal{N}(\alpha, \beta, \delta, t) \Diamond \mathcal{N}(\delta, \gamma, \gamma, s) \geq \mathcal{N}(\alpha, \beta, \gamma, t + s)$,

(xi) $\mathcal{N}(\alpha, \beta, \gamma, .) : (0, \infty) \to [0, 1]$ is continuous.

Then, the pair $(\mathcal{M}, \mathcal{N})$ is called an IGFMS on X.

Example 2.2. [12] Let $X = \Delta$. Define $a * b = \min\{a, b\}$, $a \Diamond b = \max\{a, b\}$, for all $a, b \in [0, 1]$ and $\mathcal{M}(\alpha, \beta, \gamma, t) = \left[exp\left(\frac{|\alpha-\beta|+|\beta-\gamma|+|\gamma-\alpha|}{t}\right)\right]^{-1}$ and
$\mathcal{N}(\alpha, \beta, \gamma, t) = \left[exp\left(\frac{|\alpha-\beta|+|\beta-\gamma|+|\gamma-\alpha|}{t}\right)\right]^{-1}\left[exp\left(\frac{|\alpha-\beta|+|\beta-\gamma|+|\gamma-\alpha|}{t}\right) - 1\right]$ for all $\alpha, \beta, \gamma \in X$ and $t \in (0, \infty)$. Then, $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is a IGFMS.

Example 2.3. [12] Let X be a metric space. Define $a * b = \min\{a, b\}$, $a \Diamond b = \max\{a, b\}$, for all $a, b \in [0, 1]$ and $\mathcal{M}(\alpha, \beta, \gamma, t) = \frac{kt^n}{kt^n + mD(\alpha, \beta, \gamma)}$ and $\mathcal{N}(\alpha, \beta, \gamma, t) = \frac{mD(\alpha, \beta, \gamma)}{kt^n + mD(\alpha, \beta, \gamma)}$, for all $\alpha, \beta, \gamma \in X$ and $k, m, n \in N, t \in (0, \infty)$. Then, $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is a IGFMS.

Definition 2.4. [12] Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a IGFMS. Then, a sequence $\{\mu_n\}$ in X is said to be a Cauchy sequence if
$\lim\limits_{t\to\infty}\left(\frac{1}{\mathcal{M}(\mu_n, \mu_n, \mu_{n+p}, t)} - 1\right) = 0$ and $\lim\limits_{t\to\infty} \mathcal{N}(\mu_n, \mu_n, \mu_{n+p}, t) = 0$ for all $t > 0$ and $n, p \in N$.

Definition 2.5. [12] A IGFMS $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is said to be complete if every Cauchy sequence in X converges to some point in X.

Definition 2.6. Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a IGFMS. We will say the mapping $\Xi : X \to X$ is fuzzy contractive if there exists $k \in (0, 1)$ such that $\left(\frac{1}{\mathcal{M}(\Xi\alpha, \Xi\beta, \Xi\gamma, t)} - 1\right) \leq k\left(\frac{1}{\mathcal{M}(\alpha, \beta, \gamma, t)} - 1\right)$ and $\mathcal{N}(\Xi\alpha, \Xi\beta, \Xi\gamma, t) \leq k\mathcal{N}(\alpha, \beta, \gamma, t)$ for each $\alpha, \beta, \gamma \in X$ and $t > 0$. (k is called the contractive constant of Ξ).

Lemma 2.7. Let $\{\mu_n\}$ is a sequence in a IGFMS $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ and if $\left(\frac{1}{\mathcal{M}(\mu_n, \mu_{n+1}, \mu_{n+2}, t)} - 1\right) \leq k^n\left(\frac{1}{\mathcal{M}(\mu_0, \mu_1, \mu_2, t)} - 1\right)$ and
$\mathcal{N}(\mu_n, \mu_{n+1}, \mu_{n+2}, t) \leq k^n\mathcal{N}(\mu_0, \mu_1, \mu_2, t)$where $0 < k < 1$, $n \in N$. Then, $\{\mu_n\}$ is a Cauchy sequence in X.

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Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

Theorem 2.8. [7] Let (X, d), (Y, ρ) and (Z, σ) be complete metric spaces. If T is continuous mapping of X into Y, S is a mapping of Y into Z and R is mapping of Z into X satisfying the inequalities

$$d(RST\alpha, RST\alpha^{'}) \leq c\max[d(\alpha,\alpha^{'}), d(\alpha, RST\alpha), d(\alpha^{'}, RST\alpha^{'}), \rho(T\alpha, T\alpha^{'}), \sigma(ST\alpha, ST\alpha^{'})]$$
$$\rho(TRS\beta, TRS\beta^{'}) \leq c\max[\rho(\beta,\beta^{'}), \rho(\beta, TRS\beta), \rho(\beta^{'}, TRS\beta^{'}), \sigma(S\beta, S\beta^{'}), d(RS\beta, RS\beta^{'})]$$
$$\sigma(STR\gamma, STR\gamma^{'}) \leq c\max[\sigma(\gamma,\gamma^{'}), \sigma(\gamma, STR\gamma), \sigma(\gamma^{'}, STR\gamma^{'}), d(R\gamma, R\gamma^{'}), \rho(TR\gamma, TR\gamma^{'})]$$

for all $\alpha,\alpha^{'} \in X,\ \beta,\beta^{'} \in Y$ and $\gamma,\gamma^{'} \in Z$, where $0 \leq c < 1$, then RST has unique fixed point u in X, TRS has unique fixed point υ in Y and STR has unique fixed point ω in Z. Further, $Tu = \upsilon, S\upsilon = \upsilon$, and $R\omega = u$.

3. Three Complete IGFMS

We now prove the following related fixed point theorem which improves the above theorem in IGFMS.

Theorem 3.1. Let $(X, \mathcal{M}_1, \mathcal{N}_1, *, \diamondsuit), (Y, \mathcal{M}_2, \mathcal{N}_2, *, \diamondsuit)$ and $(Z, \mathcal{M}_3, \mathcal{N}_3, *, \diamondsuit)$ be three complete IGFMS. Suppose Ξ is a continuous mapping of X into Y, Φ is a mapping of Y into Z and Δ is a mapping of Z into X satisfies the following inequalities:

$$\begin{aligned}
&\Big(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha', \Delta\Phi\Xi\alpha'', t)} - 1\Big) \\
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_1(\alpha,\alpha',\alpha'',t)} - 1\Big) - \Big(\frac{1}{\mathcal{M}_1(\alpha, \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha', t)} - 1\Big), \\
&\qquad 2\left(\frac{1}{\mathcal{M}_1(\alpha', \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha', t)} - 1\right) - \left(\frac{1}{\mathcal{M}_1(\alpha, \Delta\Phi\Xi\alpha', \alpha'', t)} - 1\right), \\
&\qquad 2\left(\frac{1}{\mathcal{M}_1(\alpha', \Delta\Phi\Xi\alpha', \alpha'', t)} - 1\right) - \left(\frac{1}{\mathcal{M}_1(\alpha, \Delta\Phi\Xi\alpha, \alpha', t)} - 1\right), \\
&\qquad \left(\frac{1}{\mathcal{M}_2(\Xi\alpha, \Xi\alpha', \Xi\alpha'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_3(\Phi\Xi\alpha, \Phi\Xi\alpha', \Phi\Xi\alpha'', t)} - 1\right)\Bigg\}
\end{aligned}$$

$$\begin{aligned}
&\mathcal{N}_1(\Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha^{'}, \Delta\Phi\Xi\alpha^{''}, t) \\
&\leq k\min\left\{\begin{array}{c}
2\,\mathcal{N}_1(\alpha,\alpha^{'},\alpha^{''},t) - \mathcal{N}_1(\alpha, \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha^{'}, t), \\
2\,\mathcal{N}_1(\alpha^{'}, \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha^{'}, t) - \mathcal{N}_1(\alpha, \Delta\Phi\Xi\alpha^{'}, \alpha^{''}, t), \\
2\,\mathcal{N}_1(\alpha^{'}, \Delta\Phi\Xi\alpha^{'}, \alpha^{''}, t) - \mathcal{N}_1(\alpha, \Delta\Phi\Xi\alpha, \alpha^{'}, t), \\
\mathcal{N}_2(\Xi\alpha, \Xi\alpha^{'}, \Xi\alpha^{''}, t), \mathcal{N}_3(\Phi\Xi\alpha, \Phi\Xi\alpha^{'}, \Phi\Xi\alpha^{''}, t)
\end{array}\right\}
\end{aligned} \tag{3.1.1}$$

$$\begin{aligned}
&\Big(\frac{1}{\mathcal{M}_2(\Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta', \Xi\Delta\Phi\beta'', t)} - 1\Big) \\
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_2(\beta,\beta',\beta'',t)} - 1\Big) - \Big(\frac{1}{\mathcal{M}_2(\beta, \Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta', t)} - 1\Big),
\end{aligned}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

$$2\left(\frac{1}{\mathcal{M}_2(\beta',\Xi\Delta\Phi\beta,\Xi\Delta\Phi\beta',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\beta,\Xi\Delta\Phi\beta',\beta'',t)}-1\right),$$
$$2\left(\frac{1}{\mathcal{M}_2(\beta',\Xi\Delta\Phi\beta',\beta'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\beta,\Xi\Delta\Phi\beta,\beta',t)}-1\right),$$
$$\left(\frac{1}{\mathcal{M}_3(\Phi\beta,\Phi\beta',\Phi\beta'',t)}-1\right),\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\beta,\Delta\Phi\beta',\Delta\Phi\beta'',t)}-1\right)\Bigg\}$$

$$\mathcal{N}_2(\Xi\Delta\Phi\beta,\Xi\Delta\Phi\beta^{'},\Xi\Delta\Phi\beta^{''},t)$$
$$\leq k\min\left\{\begin{array}{c}2\,\mathcal{N}_2(\beta,\beta^{'},\beta^{''},t)-\mathcal{N}_2(\beta,\Xi\Delta\Phi\beta,\Xi\Delta\Phi\beta^{'},t),\\ 2\,\mathcal{N}_2(\beta^{'},\Xi\Delta\Phi\beta,\Xi\Delta\Phi\beta^{'},t)-\mathcal{N}_2(\beta,\Xi\Delta\Phi\beta^{'},\beta^{''},t),\\ 2\,\mathcal{N}_2(\beta^{'},\Xi\Delta\Phi\beta^{'},\beta^{''},t)-\mathcal{N}_2(\beta,\Xi\Delta\Phi\beta,\beta^{'},t),\\ \mathcal{N}_3(\Phi\beta,\Phi\beta^{'},\Phi\beta^{''},t),\mathcal{N}_1(\Delta\Phi\beta,\Delta\Phi\beta^{'},\Delta\Phi\beta^{''},t)\end{array}\right\} \tag{3.1.2}$$

$$\left(\frac{1}{\mathcal{M}_3(\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma',\Phi\Xi\Delta\gamma'',t)}-1\right)$$
$$\leq k\max\Bigg\{2\left(\frac{1}{\mathcal{M}_3(\gamma,\gamma',\gamma'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\gamma,\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma',t)}-1\right),$$
$$2\left(\frac{1}{\mathcal{M}_3(\gamma',\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\gamma,\Phi\Xi\Delta\gamma',\gamma'',t)}-1\right),$$
$$2\left(\frac{1}{\mathcal{M}_3(\gamma',\Phi\Xi\Delta\gamma',\gamma'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\gamma,\Phi\Xi\Delta\gamma,\gamma',t)}-1\right),$$
$$\left(\frac{1}{\mathcal{M}_1(\Delta\gamma,\Delta\gamma',\Delta\gamma'',t)}-1\right),\left(\frac{1}{\mathcal{M}_2(\Xi\Delta\gamma,\Xi\Delta\gamma',\Xi\Delta\gamma'',t)}-1\right)\Bigg\}$$

$$\mathcal{N}_3(\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma^{'},\Phi\Xi\Delta\gamma^{''},t)$$
$$\leq k\min\left\{\begin{array}{c}2\,\mathcal{N}_3(\gamma,\gamma^{'},\gamma^{''},t)-\mathcal{N}_3(\gamma,\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma^{'},t),\\ 2\,\mathcal{N}_3(\gamma^{'},\Phi\Xi\Delta\gamma,\Phi\Xi\Delta\gamma^{'},t)-\mathcal{N}_3(\gamma,\Phi\Xi\Delta\gamma^{'},\gamma^{''},t),\\ 2\,\mathcal{N}_3(\gamma^{'},\Phi\Xi\Delta\gamma^{'},\gamma^{''},t)-\mathcal{N}_3(\gamma,\Phi\Xi\Delta\gamma,\gamma^{'},t),\\ \mathcal{N}_1(\Delta\gamma,\Delta\gamma^{'},\Delta\gamma^{''},t),\mathcal{N}_2(\Xi\Delta\gamma,\Xi\Delta\gamma^{'},\Xi\Delta\gamma^{''},t)\end{array}\right\} \tag{3.1.3}$$

for all $\alpha,\alpha^{'},\alpha^{''}\in X$, $\beta,\beta^{'},\beta^{''}\in Y$ and $\gamma,\gamma^{'},\gamma^{''}\in Z$, where $0\leq k<1$, then $\Delta\Phi\Xi$ has unique fixed point $u\in X$, $\Xi\Delta\Phi$ has unique fixed point $v\in Y$ and $\Phi\Xi\Delta$ has unique fixed point $w\in Z$. Further, $\Xi u=v$, $\Phi v=w$ and $\Delta w=u$.

Proof:

Let α_0 be an arbitrary point in X. Define sequences $\{\alpha_n\},\{\beta_n\}$ and $\{\gamma_n\}$ in X,Y and Z respectively by $\alpha_n=(\Delta\Phi\Xi)^n\alpha_0$, $\beta_n=\Xi\alpha_{n-1}$, $\gamma_n=\Phi\beta_n$ for $n=1,2,...$ Using inequality (3.1.2), we have,

$$\left(\frac{1}{\mathcal{M}_2(\beta_n,\beta_{n+1},\beta_{n+2},t)}-1\right)=\left(\frac{1}{\mathcal{M}_2(\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,\Xi\Delta\Phi\beta_{n+1},t)}-1\right)$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

$$
\begin{aligned}
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,t)}-1\Big),\\
&\qquad 2\Big(\frac{1}{\mathcal{M}_2(\beta_n,\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\Xi\Delta\Phi\beta_n,\beta_{n+1},t)}-1\Big),\\
&\qquad 2\Big(\frac{1}{\mathcal{M}_2(\beta_n,\Xi\Delta\Phi\beta_n,\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\Xi\Delta\Phi\beta_{n-1},\beta_n,t)}-1\Big),\\
&\qquad \Big(\frac{1}{\mathcal{M}_3(\Phi\beta_{n-1},\Phi\beta_n,\Phi\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\Delta\Phi\beta_{n-1},\Delta\Phi\beta_n,\Delta\Phi\beta_{n+1},t)}-1\Big)\Bigg\}\\
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big),\\
&\qquad 2\Big(\frac{1}{\mathcal{M}_2(\beta_n,\beta_n,\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_{n+1},\beta_{n+1},t)}-1\Big),\\
&\qquad 2\Big(\frac{1}{\mathcal{M}_2(\beta_n,\beta_{n+1},\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_n,t)}-1\Big),\\
&\qquad \Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)\Bigg\}\\
&\leq k\max\Bigg\{\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big),\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big),\\
&\qquad\qquad \Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{N}_2(\beta_n,\beta_{n+1},\beta_{n+2},t)=\mathcal{N}_2(\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,\Xi\Delta\Phi\beta_{n+1},t)\\
&\quad\leq k\min\left\{\begin{array}{c}
2\ \mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)-\mathcal{N}_2(\beta_{n-1},\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,t),\\
2\ \mathcal{N}_2(\beta_n,\Xi\Delta\Phi\beta_{n-1},\Xi\Delta\Phi\beta_n,t)-\mathcal{N}_2(\beta_{n-1},\Xi\Delta\Phi\beta_n,\beta_{n+1},t),\\
2\ \mathcal{N}_2(\beta_n,\Xi\Delta\Phi\beta_n,\beta_{n+1},t)-\mathcal{N}_2(\beta_{n-1},\Xi\Delta\Phi\beta_{n-1},\beta_n,t),\\
\mathcal{N}_3(\Phi\beta_{n-1},\Phi\beta_n,\Phi\beta_{n+1},t),\mathcal{N}_1(\Delta\Phi\beta_{n-1},\Delta\Phi\beta_n,\Delta\Phi\beta_{n+1},t)
\end{array}\right\}\\
&\quad\leq k\min\left\{\begin{array}{c}
2\ \mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)-\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t),\\
2\ \mathcal{N}_2(\beta_n,\beta_n,\beta_{n+1},t)-\mathcal{N}_2(\beta_{n-1},\beta_{n+1},\beta_{n+1},t),\\
2\ \mathcal{N}_2(\beta_n,\beta_{n+1},\beta_{n+1},t)-\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_n,t),\\
\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t),\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)
\end{array}\right\}\\
&\quad\leq k\min\Big\{\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t),\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t),\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)\Big\}. \qquad (3.1.4)
\end{aligned}
$$

Using inequality (3.1.3), we have,

$$
\begin{aligned}
&\left(\frac{1}{\mathcal{M}_3(\gamma_n,\gamma_{n+1},\gamma_{n+2},t)}-1\right)=\left(\frac{1}{\mathcal{M}_3(\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,\Phi\Xi\Delta\gamma_{n+1},t)}-1\right)\\
&\quad\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,t)}-1\Big),
\end{aligned}
$$

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Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

$$
\begin{aligned}
&2\Big(\frac{1}{\mathcal{M}_3(\gamma_n,\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_n,\gamma_{n+1},t)}-1\Big),\\
&2\Big(\frac{1}{\mathcal{M}_3(\gamma_n,\Phi\Xi\Delta\gamma_n,\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_{n-1},\gamma_n,t)}-1\Big),\\
&\Big(\frac{1}{\mathcal{M}_1(\Phi\gamma_{n-1},\Phi\gamma_n,\Phi\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\Xi\Phi\gamma_{n-1},\Xi\Phi\gamma_n,\Xi\Phi\gamma_{n+1},t)}-1\Big)\Bigg\}\\
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big),\\
&2\Big(\frac{1}{\mathcal{M}_3(\gamma_n,\gamma_n,\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_{n+1},\gamma_{n+1},t)}-1\Big),\\
&2\Big(\frac{1}{\mathcal{M}_3(\gamma_n,\gamma_{n+1},\gamma_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_n,t)}-1\Big),\\
&\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big)\Bigg\}\\
&\leq k\max\Bigg\{\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big),\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big),\\
&\qquad\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{N}_3(\gamma_n,\gamma_{n+1},\gamma_{n+2},t)=\mathcal{N}_3(\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,\Phi\Xi\Delta\gamma_{n+1},t)\\
&\leq k\min\left\{\begin{array}{c}
2\,\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)-\mathcal{N}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,t),\\
2\,\mathcal{N}_3(\gamma_n,\Phi\Xi\Delta\gamma_{n-1},\Phi\Xi\Delta\gamma_n,t)-\mathcal{N}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_n,\gamma_{n+1},t),\\
2\,\mathcal{N}_3(\gamma_n,\Phi\Xi\Delta\gamma_n,\gamma_{n+1},t)-\mathcal{N}_3(\gamma_{n-1},\Phi\Xi\Delta\gamma_{n-1},\gamma_n,t),\\
\mathcal{N}_1(\Phi\gamma_{n-1},\Phi\gamma_n,\Phi\gamma_{n+1},t),\mathcal{N}_2(\Xi\Phi\gamma_{n-1},\Xi\Phi\gamma_n,\Xi\Phi\gamma_{n+1},t)
\end{array}\right\}.\\
&\leq k\min\left\{\begin{array}{c}
2\,\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)-\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t),\\
2\,\mathcal{N}_3(\gamma_n,\gamma_n,\gamma_{n+1},t)-\mathcal{N}_3(\gamma_{n-1},\gamma_{n+1},\gamma_{n+1},t),\\
2\,\mathcal{N}_3(\gamma_n,\gamma_{n+1},\gamma_{n+1},t)-\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_n,t),\\
\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t),\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)
\end{array}\right\}\\
&\leq k\min\Big\{\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t),\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t),\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)\Big\}, \qquad (3.1.5)
\end{aligned}
$$

on using inequalities (3.1.4) and (3.1.1), we have,

$$
\begin{aligned}
&\left(\frac{1}{\mathcal{M}_1(\alpha_n,\alpha_{n+1},\alpha_{n+2},t)}-1\right)=\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,\Delta\Phi\Xi\alpha_{n+1},t)}-1\right)\\
&\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t)}-1\Big),\\
&\qquad 2\Big(\frac{1}{\mathcal{M}_1(\alpha_n,\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_n,\alpha_{n+1},t)}-1\Big),
\end{aligned}
$$

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ISSN: 1309-3452

$$2\Big(\frac{1}{\mathcal{M}_1(\alpha_n,\Delta\Phi\Xi\alpha_n,\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t)}-1\Big),$$
$$\Big(\frac{1}{\mathcal{M}_2(\Xi\alpha_{n-1},\Xi\alpha_n,\Xi\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,\Phi\Xi\alpha_{n+1},t)}-1\Big)\Bigg\}$$
$$\leq k\max\Bigg\{2\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big),$$
$$2\Big(\frac{1}{\mathcal{M}_1(\alpha_n,\alpha_n,\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_{n+1},\alpha_{n+1},t)}-1\Big),$$
$$2\Big(\frac{1}{\mathcal{M}_1(\alpha_n,\alpha_{n+1},\alpha_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_n,t)}-1\Big),$$
$$\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big)-\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big)\Bigg\}$$
$$\leq k\max\Bigg\{\Big(\frac{1}{\mathcal{M}_2(\beta_{n-1},\beta_n,\beta_{n+1},t)}-1\Big),\Big(\frac{1}{\mathcal{M}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)}-1\Big),$$
$$\Big(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)}-1\Big)\Bigg\}.$$

$$\mathcal{N}_1(\alpha_n,\alpha_{n+1},\alpha_{n+2},t)=\mathcal{N}_1(\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,\Delta\Phi\Xi\alpha_{n+1},t)$$
$$\leq k\min\left\{\begin{array}{c}2\,\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)-\mathcal{N}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t),\\ 2\,\mathcal{N}_1(\alpha_n,\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t)-\mathcal{N}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_n,\alpha_{n+1},t),\\ 2\,\mathcal{N}_1(\alpha_n,\Delta\Phi\Xi\alpha_n,\alpha_{n+1},t)-\mathcal{N}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t),\\ \mathcal{N}_2(\Xi\alpha_{n-1},\Xi\alpha_n,\Xi\alpha_{n+1},t),\mathcal{N}_3(\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,\Phi\Xi\alpha_{n+1}\end{array}\right\}$$
$$\leq k\min\left\{\begin{array}{c}2\,\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)-\mathcal{N}_1(\alpha_{n-1)},\alpha_n,\alpha_{n+1},t),\\ 2\,\mathcal{N}_1(\alpha_n,\alpha_n,\alpha_{n+1},t)-\mathcal{N}_1(\alpha_{n-1},\alpha_{n+1},\alpha_{n+1},t)\\ 2\,\mathcal{N}_1(\alpha_n,\alpha_{n+1},\alpha_{n+1},t)-\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_n,t),\\ \mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t),\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t)\end{array}\right\}$$
$$\leq k\min\Big\{\mathcal{N}_2(\beta_{n-1},\beta_n,\beta_{n+1},t),\mathcal{N}_3(\gamma_{n-1},\gamma_n,\gamma_{n+1},t),\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_{n+1},t)\Big\},\tag{3.1.6}$$

on using inequalities (3.1.4) and (3.1.5). It follows easily by induction on using inequalities (3.1.4), (3.1.5) and (3.1.6) that

$$\left(\frac{1}{\mathcal{M}_1(\alpha_n,\alpha_{n+1},\alpha_{n+2},t)}-1\right)$$
$$\leq c^{n-1}\max\left\{\left(\frac{1}{\mathcal{M}_1(\alpha_1,\alpha_2,\alpha_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_2(\beta_1,\beta_2,\beta_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\gamma_1,\gamma_2,\gamma_3,t)}-1\right)\right\},$$

$$\left(\frac{1}{\mathcal{M}_2(\beta_n,\beta_{n+1},\beta_{n+2},t)}-1\right)$$

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https://publishoa.com
ISSN: 1309-3452

$$\leq c^{n-1}\max\left\{\left(\frac{1}{\mathcal{M}_1(\alpha_1,\alpha_2,\alpha_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_2(\beta_1,\beta_2,\beta_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\gamma_1,\gamma_2,\gamma_3,t)}-1\right)\right\},$$

$$\left(\frac{1}{\mathcal{M}_3(\gamma_n,\gamma_{n+1},\gamma_{n+2},t)}-1\right)$$
$$\leq c^{n-1}\max\left\{\left(\frac{1}{\mathcal{M}_1(\alpha_1,\alpha_2,\alpha_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_2(\beta_1,\beta_2,\beta_3,t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\gamma_1,\gamma_2,\gamma_3,t)}-1\right)\right\},$$

$$\mathcal{N}_1(\alpha_n,\alpha_{n+1},\alpha_{n+2},t)\leq c^{n-1}\min\left\{\mathcal{N}_1(\alpha_1,\alpha_2,\alpha_3,t),\mathcal{N}_2(\beta_1,\beta_2,\beta_3,t),\mathcal{N}_3(\gamma_1,\gamma_2,\gamma_3,t)\right\},$$
$$\mathcal{N}_2(\beta_n,\beta_{n+1},\beta_{n+2},t)\leq c^{n-1}\min\left\{\mathcal{N}_1(\alpha_1,\alpha_2,\alpha_3,t),\mathcal{N}_2(\beta_1,\beta_2,\beta_3,t),\mathcal{N}_3(\gamma_1,\gamma_2,\gamma_3,t)\right\},$$
$$\mathcal{N}_3(\gamma_n,\gamma_{n+1},\gamma_{n+2},t)\leq c^{n-1}\min\left\{\mathcal{N}_1(\alpha_1,\alpha_2,\alpha_3,t),\mathcal{N}_2(\beta_1,\beta_2,\beta_3,t),\mathcal{N}_3(\gamma_1,\gamma_2,\gamma_3,t)\right\}.$$

Since $c<1$, it follows that $\{\alpha_n\},\{\beta_n\}$ and $\{\gamma_n\}$ are Cauchy sequences with limits u,v and w in X,Y and Z respectively. Since Ξ and Φ are continuous, we have,

$$\lim_{n\to\infty}\beta_n=\lim_{n\to\infty}T\alpha_n=Tu=v\quad\text{and}\quad\lim_{n\to\infty}\gamma_n=\lim_{n\to\infty}\Phi\beta_n=\Phi v=w.$$

Using inequality (3.1.1) again, we have,

$$\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t)}-1\right)$$
$$\leq k\max\left\{2\left(\frac{1}{\mathcal{M}_1(u,\alpha_{n-1},\alpha_n,t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},t)}-1\right),\right.$$
$$2\left(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t)}-1\right),$$
$$2\left(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,\alpha_{n-1},t)}-1\right),$$
$$\left.\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\alpha_{n-1},\Xi\alpha_n,t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,t)}-1\right)\right\}$$
$$\leq k\max\left\{2\left(\frac{1}{\mathcal{M}_1(u,\alpha_{n-1},\alpha_n,t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,\alpha_n,t)}-1\right),\right.$$
$$2\left(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\Delta\Phi\Xi u,\alpha_n,t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\alpha_n,\alpha_n,t)}-1\right),$$
$$2\left(\frac{1}{\mathcal{M}_1(\alpha_{n-1},\alpha_n,\alpha_n,t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,\alpha_{n-1},t)}-1\right),$$
$$\left.\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\alpha_{n-1},\Xi\alpha_n,t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,t)}-1\right)\right\}.$$

$$\mathcal{N}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},\Delta\Phi\Xi\alpha_n,t)$$

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https://publishoa.com
ISSN: 1309-3452

$$\leq k\min\left\{\begin{array}{c} 2\,\mathcal{N}_1(u,\alpha_{n-1},\alpha_n,t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},t),\\ 2\,\mathcal{N}_1(\alpha_{n-1},\Delta\Phi\Xi u,\Delta\Phi\Xi\alpha_{n-1},t)-\mathcal{N}_1(u,\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t),\\ 2\,\mathcal{N}_1(\alpha_{n-1}.\Delta\Phi\Xi\alpha_{n-1},\alpha_n,t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,\alpha_{n-1},t),\\ \mathcal{N}_2(\Xi u,\Xi\alpha_{n-1},\Xi\alpha_n,t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,t)\end{array}\right\}$$

$$\leq k\min\left\{\begin{array}{c} 2\,\mathcal{N}_1(u,\alpha_{n-1},\alpha_n,t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,\alpha_n,t),\\ 2\,\mathcal{N}_1(\alpha_{n-1},\Delta\Phi\Xi u,\alpha_n,t)-\mathcal{N}_1(u,\alpha_n,\alpha_n,t),\\ 2\,\mathcal{N}_1(\alpha_{n-1},\alpha_n,\alpha_n,t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,\alpha_{n-1},t),\\ \mathcal{N}_2(\Xi u,\Xi\alpha_{n-1},\Xi\alpha_n,t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi\alpha_{n-1},\Phi\Xi\alpha_n,t)\end{array}\right\}.$$

Since Φ and Ξ are continuous, it follows on letting $n\to\infty$ that

$$\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,u,u,t)}-1\right)\leq k\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,u,u,t)}-1\right),$$
$$\mathcal{N}_1(\Delta\Phi\Xi u,u,u,t)\leq k\,\mathcal{N}_1(\Delta\Phi\Xi u,u,u,t).$$

Thus, $\Delta\Phi\Xi u=u$, since $k<1$ and so u is a fixed point of $\Delta\Phi\Xi$. We therefore have $\Xi\Delta\Phi v=\Xi\Delta\Phi\Xi u=\Xi u=v$ and so $\Phi\Xi\Delta w=\Phi\Xi\Delta\Phi v=\Phi v=w$.

Hence, v and w are fixed points of $\Xi\Delta\Phi$ and $\Phi\Xi\Delta$ respectively.

We now prove the uniqueness of the fixed point u. Suppose that $\Delta\Phi\Xi$ has a second fixed point u.

Then, using inequality (3.1.1), we have,

$$\begin{aligned}
\Big(\frac{1}{\mathcal{M}_1(u,u',u',t)}-1\Big) &= \Big(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u',\Delta\Phi\Xi u'',t)}-1\Big)\\
&\leq k\max\bigg\{2\left(\frac{1}{\mathcal{M}_1(u,u',u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,\Delta\Phi\Xi u',t)}-1\right),\\
&\quad 2\left(\frac{1}{\mathcal{M}_1(u',\Delta\Phi\Xi u,\Delta\Phi\Xi u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u',u'',t)}-1\right),\\
&\quad 2\left(\frac{1}{\mathcal{M}_1(u',\Delta\Phi\Xi u',u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,u',t)}-1\right),\\
&\quad \left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u',t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)\bigg\}\\
&\leq k\max\bigg\{2\left(\frac{1}{\mathcal{M}_1(u,u',u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,u',t)}-1\right),\\
&\quad 2\left(\frac{1}{\mathcal{M}_1(u',\Delta\Phi\Xi u,u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,u',u'',t)}-1\right),\\
&\quad 2\left(\frac{1}{\mathcal{M}_1(u',u',u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_1(u,\Delta\Phi\Xi u,u',t)}-1\right),\\
&\quad \left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)\bigg\}\\
&\leq k\max\bigg\{\left(\frac{1}{\mathcal{M}_1(\Xi u,\Xi u',\Xi u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)\bigg\}.
\end{aligned}$$

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https://publishoa.com
ISSN: 1309-3452

$$\begin{aligned}
\mathcal{N}_1(u,u',u',t) &= \mathcal{N}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u',\Delta\Phi\Xi u'',t)\\
&\le k\min\left\{\begin{array}{c} 2\,\mathcal{N}_1(u,u',u',t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,\Delta\Phi\Xi u',t)\\ 2\,\mathcal{N}_1(u',\Delta\Phi\Xi u,\Delta\Phi\Xi u',t)-\mathcal{N}_1(u,\Delta\Phi\Xi u',u'',t),\\ 2\,\mathcal{N}_1(u',\Delta\Phi\Xi u',u'',t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,u',t),\\ \mathcal{N}_2(\Xi u,\Xi u',\Xi u',t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)\end{array}\right\}\\
&\le k\min\left\{\begin{array}{c} 2\,\mathcal{N}_1(u,u',u'',t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,u',t),\\ 2\,\mathcal{N}_1(u',\Delta\Phi\Xi u,u',t)-\mathcal{N}_1(u,u',u'',t),\\ 2\,\mathcal{N}_1(u',u',u'',t)-\mathcal{N}_1(u,\Delta\Phi\Xi u,u',t),\\ \mathcal{N}_2(\Xi u,\Xi u',\Xi u'',t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)\end{array}\right\}\\
&\le k\min\left\{\mathcal{N}_1(\Xi u,\Xi u',\Xi u'',t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)\right\}.
\end{aligned}$$

Further, using inequality (3.1.2), we have,

$$\begin{aligned}
&\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u'',t)}-1\right)=\left(\frac{1}{\mathcal{M}_2(\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u',\Xi\Delta\Phi\Xi u'',t)}-1\right)\\
&\le k\max\Bigg\{2\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u',t)}-1\right),\\
&\qquad 2\left(\frac{1}{\mathcal{M}_2(\Xi u',\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\Delta\Phi\Xi u',\Xi u'',t)}-1\right),\\
&\qquad 2\left(\frac{1}{\mathcal{M}_2(\Xi u',\Xi\Delta\Phi\Xi u',\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi u',t)}-1\right),\\
&\qquad \left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u',\Delta\Phi\Xi u'',t)}-1\right)\Bigg\}\\
&\le k\max\Bigg\{2\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u,\Xi u',t)}-1\right),\\
&\qquad 2\left(\frac{1}{\mathcal{M}_2(\Xi u',\Xi u,\Xi u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi u',\Xi u'',t)}-1\right),\\
&\qquad 2\left(\frac{1}{\mathcal{M}_2(\Xi u',\Xi u',\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi u',t)}-1\right),\\
&\qquad \left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_1(u,u',u'',t)}-1\right)\Bigg\}\\
&\le k\max\left\{\left(\frac{1}{\mathcal{M}_1(u,u',u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)\right\}.
\end{aligned}$$

$$\mathcal{N}_2(\Xi u,\Xi u',\Xi u'',t)=\mathcal{N}_2(\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u',\Xi\Delta\Phi\Xi u'',t)$$

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Volume 13, No. 3, 2022, p. 1665-1678
https://publishoa.com
ISSN: 1309-3452

$$\leq k\min\left\{\begin{array}{c} 2\mathcal{N}_2(\Xi u,\Xi u^{'},\Xi u^{"},t)-\mathcal{N}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u^{'},t),\\ 2\ \mathcal{N}_2(\Xi u^{'},\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u^{'},t)-\mathcal{N}_2(\Xi u,\Xi\Delta\Phi\Xi u^{'},\Xi u^{''},t),\\ 2\ \mathcal{N}_2(\Xi u^{'},\Xi\Delta\Phi\Xi u^{'},\Xi u^{''},t)-\mathcal{N}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi u^{'},t),\\ \mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t),\mathcal{N}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u^{'},\Delta\Phi\Xi u'',t)\end{array}\right\}$$

$$\leq k\min\left\{\begin{array}{c} 2\ \mathcal{N}_2(\Xi u,\Xi u^{'},\Xi u^{''},t)-\mathcal{N}_2(\Xi u,\Xi u,\Xi u^{'},t),\\ 2\ \mathcal{N}_2(\Xi u^{'},\Xi u,\Xi u^{'},t)-\mathcal{N}_2(\Xi u,\Xi u^{'},\Xi u^{''},t),\\ 2\ \mathcal{N}_2(\Xi u^{'},\Xi u^{'},\Xi u^{''},t)-\mathcal{N}_2(\Xi u,\Xi\Delta\Phi\Xi u,\Xi u^{'},t),\\ \mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t),\mathcal{N}_1(u,u^{'},u^{''},t)\end{array}\right\}$$

$$\leq k\min\left\{\mathcal{N}_1(u,u^{'},u^{''},t),\mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t)\right\}.$$

Hence, we have $\left(\frac{1}{\mathcal{M}_1(u,u^{'},u^{''},t)}-1\right)\leq k\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t)}-1\right)$ and
$\mathcal{N}_1(u,u^{'},u^{''},t)\leq k\ \mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t)$.

Finally on using inequality (3.1.3), we have

$$\begin{aligned}
&\left(\frac{1}{\mathcal{M}_1(u,u',u",t)}-1\right)\\
&\leq k\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)\\
&\leq k\left(\frac{1}{\mathcal{M}_3(\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u',\Phi\Xi\Delta\Phi\Xi u'',t)}-1\right)\\
&\leq k^2\max\left\{2\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi u',\Phi\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u',t)}-1\right),\right.\\
&\qquad 2\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u',\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u',\Phi\Xi u'',t)}-1\right),\\
&\qquad 2\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u',\Phi\Xi\Delta\Phi\Xi u',\Phi\Xi u'',t)}-1\right)-\left(\frac{1}{\mathcal{M}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi u',t)}-1\right),\\
&\qquad \left.\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u',\Delta\Phi\Xi u'',t)}-1\right),\left(\frac{1}{\mathcal{M}_2(\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u',\Xi\Delta\Phi\Xi u'',t)}-1\right)\right\}\\
&\leq k^2\left(\frac{1}{\mathcal{M}_1(u,u',u'',t)}-1\right).
\end{aligned}$$

$$\begin{aligned}
\mathcal{N}_1(u,u^{'},u^{''},t)&\leq k\ \mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u'',t)\\
&\leq k\ \mathcal{N}_3(\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u^{'},\Phi\Xi\Delta\Phi\Xi u^{''},t)\\
&\leq k^2\min\left\{\begin{array}{c} 2\ \mathcal{N}_3(\Phi\Xi u,\Phi\Xi u^{'},\Phi\Xi u^{''},t)-\mathcal{N}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u^{'},t),\\ 2\ \mathcal{N}_3(\Phi\Xi u^{'},\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u^{'},t)-\mathcal{N}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u^{'},\Phi\Xi u^{''},t),\\ 2\ \mathcal{N}_3(\Phi\Xi u^{'},\Phi\Xi\Delta\Phi\Xi u^{'},\Phi\Xi u^{''},t)-\mathcal{N}_3(\Phi\Xi u,\Phi\Xi\Delta\Phi\Xi u,\Phi\Xi u^{'},t),\\ \mathcal{N}_1(\Delta\Phi\Xi u,\Delta\Phi\Xi u^{'},\Delta\Phi\Xi u^{''},t),\mathcal{N}_2(\Xi\Delta\Phi\Xi u,\Xi\Delta\Phi\Xi u^{'},\Xi\Delta\Phi\Xi u^{''},t)\end{array}\right\}\\
&=k^2\ \mathcal{N}_1(u,u^{'},u^{"},t).
\end{aligned}$$

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https://publishoa.com
ISSN: 1309-3452

Since $k < 1$, it follows that $u = u^{'} = u^{''}$ and the uniqueness of u follows. Similarly, it can be proved that v is the unique fixed point of $\Xi\Delta\Phi$ and w is the unique fixed point of $\Phi\Xi\Delta$. We finally prove that we also have $\Delta w = u$. To do this, note that $\Delta w = \Delta(\Phi\Xi\Delta w) = \Delta\Phi\Xi(\Delta w)$ and so Δw is a fixed point of $\Delta\Phi\Xi$. Since u is the unique fixed point of $\Delta\Phi\Xi$, if follows that $\Delta w = u$. This completes the proof of the theorem.

Corollary 3.2. Let $(X, \mathcal{M}_1, \mathcal{N}_1, *, \Diamond), (Y, \mathcal{M}_2, \mathcal{N}_2, *, \Diamond)$ and $(Z, \mathcal{M}_3, \mathcal{N}_3, *, \Diamond)$ be three complete IGFMS. Suppose Ξ is a continuous mapping of X into Y, Φ is a mapping of Y into Z and Δ is a mapping of Z into X satisfying the following inequalities:

$$\begin{aligned}&\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha', \Delta\Phi\Xi\alpha'', t)} - 1\right)\\ &\qquad \le k \max\left\{ \left(\frac{1}{\mathcal{M}_1(\alpha, \alpha', \alpha'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_1(\alpha', \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha', t)} - 1\right),\right.\\ &\qquad\qquad \left(\frac{1}{\mathcal{M}_1(\alpha', \Delta\Phi\Xi\alpha', \alpha'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_2(\Xi\alpha, \Xi\alpha', \Xi\alpha'', t)} - 1\right),\\ &\qquad\qquad\qquad \left.\left(\frac{1}{\mathcal{M}_3(\Phi\Xi\alpha, \Phi\Xi\alpha', \Phi\Xi\alpha'', t)} - 1\right)\right\}\end{aligned}$$

$$\mathcal{N}_1(\Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha^{'}, \Delta\Phi\Xi\alpha^{''}, t) \le k \min\left\{\begin{array}{c}\mathcal{N}_1(\alpha, \alpha^{'}, \alpha^{''}, t), \mathcal{N}_1(\alpha^{'}, \Delta\Phi\Xi\alpha, \Delta\Phi\Xi\alpha^{'}, t),\\ \mathcal{N}_1(\alpha^{'}, \Delta\Phi\Xi\alpha^{'}, \alpha^{''}, t), \mathcal{N}_2(\Xi\alpha, \Xi\alpha^{'}, \Xi\alpha^{''}, t),\\ \mathcal{N}_3(\Phi\Xi\alpha, \Phi\Xi\alpha^{'}, \Phi\Xi\alpha^{''}, t)\end{array}\right\}$$

$$\begin{aligned}&\left(\frac{1}{\mathcal{M}_2(\Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta', \Xi\Delta\Phi\beta'', t)} - 1\right)\\ &\qquad \le k \max\left\{ \left(\frac{1}{\mathcal{M}_2(\beta, \beta', \beta'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_2(\beta', \Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta', t)} - 1\right),\right.\\ &\qquad\qquad \left(\frac{1}{\mathcal{M}_2(\beta', \Xi\Delta\Phi\beta', \beta'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_3(\Phi\beta, \Phi\beta', \Phi\beta'', t)} - 1\right),\\ &\qquad\qquad\qquad \left.\left(\frac{1}{\mathcal{M}_1(\Delta\Phi\beta, \Delta\Phi\beta', \Delta\Phi\beta'', t)} - 1\right)\right\}\end{aligned}$$

$$\mathcal{N}_2(\Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta^{'}, \Xi\Delta\Phi\beta^{''}, t) \le k \min\left\{\begin{array}{c}\mathcal{N}_2(\beta, \beta^{'}, \beta^{''}, t), \mathcal{N}_2(\beta^{'}, \Xi\Delta\Phi\beta, \Xi\Delta\Phi\beta^{'}, t),\\ \mathcal{N}_2(\beta^{'}, \Xi\Delta\Phi\beta^{'}, \beta^{''}, t), \mathcal{N}_3(\Phi\beta, \Phi\beta^{'}, \Phi\beta^{''}, t),\\ \mathcal{N}_1(\Delta\Phi\beta, \Delta\Phi\beta^{'}, \Delta\Phi\beta^{''}, t)\end{array}\right\}$$

$$\left(\frac{1}{\mathcal{M}_3(\Phi\Xi\Delta\gamma, \Phi\Xi\Delta\gamma', \Phi\Xi\Delta\gamma'', t)} - 1\right)$$

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$$\leq k \max \Bigg\{ \left(\frac{1}{\mathcal{M}_3(\gamma, \gamma', \gamma'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_3(\gamma', \Phi\Xi\Delta\gamma, \Phi\Xi\Delta\gamma', t)} - 1\right), \left(\frac{1}{\mathcal{M}_3(\gamma', \Phi\Xi\Delta\gamma', \gamma'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_1(\Delta\gamma, \Delta\gamma', \Delta\gamma'', t)} - 1\right), \left(\frac{1}{\mathcal{M}_2(\Xi\Delta\gamma, \Xi\Delta\gamma', \Xi\Delta\gamma'', t)} - 1\right) \Bigg\}$$

$$\mathcal{N}_3(\Phi\Xi\Delta\gamma, \Phi\Xi\Delta\gamma^{'}, \Phi\Xi\Delta\gamma^{''}, t) \leq k \min \left\{ \begin{array}{c} \mathcal{N}_3(\gamma, \gamma^{'}, \gamma^{''}, t), \mathcal{N}_3(\gamma^{'}, \Phi\Xi\Delta\gamma, \Phi\Xi\Delta\gamma^{'}, t), \\ \mathcal{N}_3(\gamma^{'}, \Phi\Xi\Delta\gamma^{'}, \gamma^{''}, t), \mathcal{N}_1(\Delta\gamma, \Delta\gamma^{'}, \Delta\gamma^{''}, t), \\ \mathcal{N}_2(\Xi\Delta\gamma, \Xi\Delta\gamma^{'}, \Xi\Delta\gamma'', t) \end{array} \right\},$$

for all $\alpha, \alpha^{'}, \alpha^{''} \in X$, $\beta, \beta^{'}, \beta^{''} \in Y$ and $\gamma, \gamma^{'}, \gamma^{''} \in Z$, where $0 \leq k < 1$, then $\Delta\Phi\Xi$ has unique fixed point $\mu \in X$, $\Xi\Delta\Phi$ has unique fixed point $v \in Y$ and $\Phi\Xi\Delta$ has unique fixed point $w \in Z$. Further, $\Xi\mu = v$, $\Phi v = w$ and $\Delta w = \mu$.

4. Conclusion

In this paper, we have proved common fixed point theorems for three complete intuitionistic generalized fuzzy metric spaces.

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