JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1785-1790
https://publishoa.com
ISSN: 1309-3452

Solving Game Theory Using Reverse Order Pentagonal Fuzzy Numbers

1R. Gajalakshmi

[1]Research Scholar, Department of Mathematics,
Vels Institute of Science, Technology & Advanced Studies , Chennai – 117, India.
E-mail: gajaharish1987@gmail.com

2 G. Charles Rabinson*

Assistant Professor, Department of Mathematics,
Vels Institute of Science, Technology & Advanced Studies , Chennai – 117, India.
E-mail: gjcrm28@gmail.com/charles.sbs@velsuniv.ac.in

Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

Abstract: In this research paper, reverse order pentagonal fuzzy numbers are introduced to solve game theory imprecise entries in its payoff matrix. All these imprecise entries are assumed to be reverse order pentagonal fuzzy numbers. Also the proposed approach provides fuzzy optimal solution of the fuzzy game converting to crisp values. The ideas are experimented by some numerical examples.

Keywords: Fuzzy numbers, Pentagonal fuzzy numbers, Reverse order pentagonal fuzzy numbers, Fuzzy game problem, Crisp value, Fuzzy ranking.

1. **1. Introduction**

[7] Game theory is a method for the study of decision-making in situations of conflicts and sometimes cooperation. Game theory provides a mathematical process for selecting an optimal strategy.

It was developed to quantify, model and explain human behavior under conflicts between individuals and public interests. A player in a game is an autonomous decision-making unit.

A strategy is a decision rule that specifies how the player will act in every possible circumstance. The mathematical treatment of the Game Theory was made available in 1944 by John Von Newman through their book "Theory of Games and Economic Behavior". The Von Newman's approach to solve the Game Theory problems was based on the principle of best out of the worst i.e., he utilized the idea of minimization of the maximum losses.

In a game problem each player's attempts to take best decision by selection various strategies from the set of available strategies. The traditional game theory assumes the existence of exact payoffs to solve competitive situations. Game theory is applicable to situations such as two players struggling to win at chess, candidates fighting an election, firms struggling to maintain their market shares etc,..

Most of the competitive game theory problems can be handled by this principle. However, in real life situations, the information available is of imprecise nature and there is an inherent degree of vagueness or uncertainty present in the system under consideration. Hence the classical mathematical techniques may not be useful to formulate and solve the real world problems. In such situations, the fuzzy sets introduced by Zadeh in 1965 provide effective and efficient tools and techniques to handle these problems.

[9] Ranking fuzzy numbers plays an important role in decision making process. It was firstly proposed by Zadeh. Bellman and Zadeh elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. Jain was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager used the concept of centroids in the ranking of fuzzy numbers.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1785-1790
https://publishoa.com
ISSN: 1309-3452

[12] D.Dubois and H.Prade has defined fuzzy number as a fuzzy subset of the real line So far fuzzy numbers like triangular fuzzy numbers, trapezoidal fuzzy numbers, Pentagonal fuzzy numbers, Hexagonal, Octagonal , pyramid fuzzy numbers and Diamond fuzzy number have been introduced with its membership functions. These numbers have got many applications like non-linear equations, risk analysis and reliability. Many operations were done using fuzzy numbers.

Madani reviewed applicability of game theory to water resources management and conflict resolution through a series of non-cooperative water resource games. His paper illustrates the dynamic structure of water resource problems and the importance of considering the game's evolution path while studying such problems. In this paper, we have proposed a new approach based on the principle of dominance for the fuzzy optimal solution of the pentagonal fuzzy valued game without converting to its equivalent crisp form.

2.1 Fuzzy set: Let X = {x} denote a collection of objects denoted generically by x. Then a fuzzy set Å in X is a set of ordered pairs Å = {(x, μ Å (x); x ∈ X} where μ Å (x) is termed as the grade of membership of x in A and μ Å : X → M is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1. A is non fuzzy and its membership function becomes identical with the characteristic function of a non fuzzy set.

2.2 Normal set: A Fuzzy set˜ of universe set X is normal if and only if x ∈ X sup μÅ(x) = 1.

2.3 Convex set: [10]A fuzzy set Å in universal set X is called convex if and only if

μ Å $(x_1\lambda + (1-\lambda)x_2) \geq$ min [μ Å (x_1), μ Å (x_2)] for all x_1 , x_2 ∈ X and $\lambda \epsilon$ [0,1].

2.4: Fuzzy Numbers: A Fuzzy number is a fuzzy set on the real line R, must satisfy the following conditions.

(i) μ Å (x) is piecewise continuous.

(ii) There exists atleast x ∈R with μ Å (x) = 1.

(iii) Å must be convex and normal.

3.1:Pentagonal Fuzzy Numbers: A fuzzy set = (p, q, r, s,t) is said to pentagonal fuzzy number if its membership function is given by where p≤ q ≤ r ≤ s≤t

$$\mu \text{ Å } (x) = \begin{cases} 0, & for\ x < p \\ \frac{x-p}{q-p}, & p \leq x \leq q \\ \frac{x-q}{r-q}, & q \leq x \leq r \\ 1, & x = r \\ \frac{s-x}{s-r}, & r \leq x \leq s \\ \frac{t-x}{t-s}, & s \leq x \leq t \\ 0, & for\ x > t \end{cases}$$

3.2:Reverse order Pentagonal fuzzy number: A fuzzy set A_{RP}= {-p,-q,0,r,s}is said to be Reverse Order Pentagonal fuzzy number(ROPFN) if its membership function is given by

$$\mu_{A_{RP}}(x) = \begin{cases} 1, & r \leq x \leq -p \\ \frac{-x}{p}, & -p \leq x \leq -q \\ \frac{-x}{q}, & -q \leq x \leq 0 \\ \frac{x}{r}, & 0 \leq x \leq r \\ \frac{x}{s}, & r \leq x \leq s \\ 1, & x \geq s \end{cases}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1785-1790
https://publishoa.com
ISSN: 1309-3452

3.3: Ranking of Pentagonal Fuzzy Numbers:

[13] Basic Concept of Ranking Fuzzy Numbers The concept of ranking fuzzy numbers is very important for decision making problems. Researchers have presented many different reasons for finding the ranking of fuzzy numbers. In this section we find a new concept for finding the ranking of PFNs. If we consider the average of these, we can obtain the new ranking as

$$R(A_{RP}) = \left(\frac{2p+9q+2r+9s+2t}{24}\right)$$

3.4: Mathematical Formulation of Fuzzy Game Problem:

Illustrate two-person zero-sum game in which all the entries in the payoff matrix are Icosidodecahedron fuzzy numbers. Let the player P has ‘ i’ strategies and player Q has ‘j’ strategies. Here it is simulated that each player has to choose from amongst the pure strategies. Player P is always assumed to be winner and player Q is always assumed to be a looser. The payoff i×j is

$$P=\begin{bmatrix} p_{11} & p_{12}\cdots & p_{1i} \\ p_{21} & p_{22}\cdots & p_{2i}. \\ p_{31} & p_{32}\cdots & p_{3i} \\ & \vdots & \\ & \vdots & \\ p_{j1}p_{j2} & \cdots\cdots\cdots . & p_{ji} \end{bmatrix}$$

3.5: Algorithm for Solving a Game Problem Using Fuzzy:

Step1: First, we have to examine whether a saddle point will exist or not. If it exists in the given problem, then we need to find it in a direct way. However, if it does not exist, then we need to follow the second step.

Step2: Compare column strategies.

(i) In the given pay-off matrix, if the elements of Column A ≤ elements of Column B, that is, Column A strategy will fully dominate over column B strategy, then, according to the rule we need to delete column B strategy from the given pay-off matrix.

(ii) In the given pay-off matrix, we tally each column strategy with all other column strategies and omit high strategies as far as possible.

Step3: Compare row strategies.

(i) In the given pay-off matrix, if the elements of Row A ≥ elements of Row B, Row A strategy will dominate over Row B strategy. So, omit Row B strategy from the given payoff matrix.

(ii) In the given pay-off matrix, we tally each row strategy with all possible row strategies and omit low strategies as far as possible.

(iii) The game may decrease to a single cell giving an order about the value of the game and

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1785-1790
https://publishoa.com
ISSN: 1309-3452

optimal master plan of the players. If not, then moves to step 4.

Step4: The dominance rule should not be based on the dominance of pure strategies only. A

given approach can be dominated if it gives us a poor result to an usual of two or more other pure

master plan.

3.6: Saddle point: If in a game, the max-min value equals the mini-max value, then the game is said to

have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is called the crisp game value of the game matrix.

Solution of all i×j Matrix Game [14] Consider the general game matrix P=$[P_{ij}]$. To solve this game we proceed as follows:

- Test for a saddle point.
- If there is no saddle point, solve by finding equalizing strategies.

The Optimal mixed strategies for player A = (p_1; p_2; p_3) and for player B = (q_1; q_2; q_3), where

$$p_1 = \left(\frac{p_{22}-p_{12}}{(p_{11}+p_{22})-(p_{12}+p_{21})}\right) \quad , p_2 = 1- p_1 \qquad\qquad q_1 = \left(\frac{p_{22}-p_{21}}{(p_{11}+p_{22})-(p_{12}+p_{21})}\right) \quad , q_2 = 1- q_1$$

$$V = \left(\frac{(p_{11}\times p_{22})-(p_{12}\times p_{21}}{(p_{11}+p_{22})-(p_{12}+p_{21})}\right)$$

Example 1. Consider the following Reverse Order Pentagonal fuzzy game problem.

Player B

Player A
$$\begin{pmatrix} (-5,-4,2,4,5) & (-13,-10,5,10,13) & (-12,-11,9,11,12) \\ (-6,-5,3,5,6) & (-4,-2,1,2,4) & (-15,-11,8,-11,15) \\ (-7,-6,5,6,7) & (-8,-7,4,7,8) & (-9,-8,7,8,9) \end{pmatrix}$$

Solution: By definition of reverse order pentagonal fuzzy number Å is calculated as

Step 1: Convert the given reverse order pentagonal fuzzy problem into a crisp value problem

$a_{11} = (-5,-4,2,4,5)$	R(a_{11}) = $\left(\frac{2(-5)+9(-4)+2(2)+9(4)+2(5)}{24}\right)$ = 0.167
$a_{12} = (-13,-10,5,10,13)$	R(a_{12}) = $\left(\frac{2(-13)+9(-10)+2(5)+9(10)+2(13)}{24}\right)$ = 0.416
$a_{13} = (-12,-11,9,11,12)$	R(a_{13}) = $\left(\frac{2(-12)+9(-11)+2(9)+9(11)+2(12)}{24}\right)$ = 0.750
$a_{21} = (-6,-5,3,5,6)$	R(a_{21}) = $\left(\frac{2(-6)+9(-5)+2(3)+9(5)+2(6)}{24}\right)$ = 0.250
$a_{22} = (-4,-2,1,2,4)$	R(a_{22}) = $\left(\frac{2(-4)+9(-2)+2(1)+9(2)+2(4)}{24}\right)$ = 0.083

a_{23} = $(\mathbf{-15,-11,8,-11,15})$	R(a_{23}) = $(\frac{2(-15)+9(-11)+2(8)+9(11)+2(15)}{24})$ = 0.667
a_{31} = $(\mathbf{-7,-6,5,6,7})$	R(a_{31}) = $(\frac{2(-7)+9(-6)+2(5)+9(6)+2(7)}{24})$ = 0.416
a_{32} = $(\mathbf{-8,-7,4,7,8})$	R(a_{32}) = $(\frac{2(-8)+9(-7)+2(4)+9(7)+2(8)}{24})$ = 0.333
a_{33} = $(\mathbf{-9,-8,7,8,9})$	R(a_{33}) = $(\frac{2(-9)+9(-8)+2(7)+9(8)+2(9)}{24})$ = 0.583

Step 2: The pay-off matrix is

Player B

Player A $\begin{pmatrix} \mathbf{0.167} & \mathbf{0.416} & \mathbf{0.750} \\ \mathbf{0.250} & \mathbf{0.083} & \mathbf{0.667} \\ \mathbf{0.416} & \mathbf{0.333} & \mathbf{0.583} \end{pmatrix}$

Step 3: There is no saddle point since Max (mini) is not equal to Min (Max).

Step 4: Using dominance the pay-off matrix

Player B

Player A $\begin{pmatrix} \mathbf{0.167} & \mathbf{0,416} & \mathbf{0.750} \\ \mathbf{0.250} & \mathbf{0.083} & \mathbf{0.667} \\ \mathbf{0.416} & \mathbf{0.333} & \mathbf{0.583} \end{pmatrix}$

Step:5: The required pay-off matrix becomes

Player B

Player A $\left(\begin{pmatrix} \mathbf{0.167} & \mathbf{0.416} \\ \mathbf{0.416} & \mathbf{0.333} \end{pmatrix}\right)$

Here p_{11} = 0.167, p_{12} = 0.416, p_{21} = 0.416 & p_{22} = 0.333

p_1 = 0.25 , p_3 = 0.75 and q_1 = 0.25, q_2 = 0.75

V = 0.35

Reference:

1. Bompard E, Carpaneto E, Ciwei G, Napoli R, Benini M, Gallanti Mand Migliavacca G, 2008, A game theory simulator for assessing the performances of competitive electricity markets Electric Power Systems Research 78 (2) pp 217- 227.
2. Campos L, Fuzzy linear programming models to solve fuzzy matrix games, 1989, Fuzzy Sets and Systems, 32(3)pp 275-289.
3. Charilas D E, and Panagopoulos A D, 2010, A survey on game theory applications in wireless Networks Computer Networks, 54 (18) pp 3421- 3430.

4. Fiestras-Janeiro M G, Garcia-Jurado I, Meca A and Mosquera M A, 2011, Cooperative game Theory and inventory management European Journal of Operational Research, 210 (3) pp 459- 466.
5. Madani K, 2010, Game theory and water resources Journal of Hydrology, 381 (3-4)pp 225- 238.
6. Sakawa M and Nishizaki I, 1994, Max-min solution for fuzzy multi-objective matrix games
7. G. Krishnaveni and K.Ganesan, 2018, “A new approach for the solution of fuzzy games”, National Conference on Mathematical Techniques and its Applications (NCMTA 18).
8. Hakan Soner Aplak and Orhan T¨urkbey, 2013, “Fuzzy logic based game theory applications in multi-criteria decision making process”, Journal of Intelligent & Fuzzy Systems 25: 359–371.
9. U.Jayalakshmi, K.Mohana, R.Jansi , 2018, “Optimal solution of Fuzzy Game Problem by using Hexadecagonal Fuzzy Numbers”, IJIRT, Volume 5 Issue 7 ISSN: 2349-6002.
10. S. Narmatha, E. Glory Bebina, and R. Vishnu Priyaa, 2020, “Solving Game Problem Using Icosikaitetragonal Fuzzy Numbers”, The International journal of analytical and experimental modal analysis, ISSN NO: 0886-9367.
11. Pathinathan. T., Ponnivalavan.K. and Mike Dison. E, 2015, “Different Types of Fuzzy Numbers and Certain Properties”,Journal of Computer and Mathematical Sciences,Vol.6(11),631-651.
12. Pathinathan. T and Ponnivalavan.K., 2015, “Reverse order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers”, Annals of Pure and Applied Mathematics, Vol. 9, No. 1, 2015, 107-117.
13. Avishek Chakraborty ,Sankar Prasad Mondal ,Shariful Alam ,Ali Ahmadian , Norazak Senu , Debashis De and Soheil Salahshour , “The Pentagonal Fuzzy Number:Its Different Representations, Properties, Ranking,Defuzzification and Application in Game Problems”, Symmetry 2019, 11, 248.
14. J.Jon Arockiaraj and N.Sivasankari, “Solution of Fuzzy Game Problem Using Hexagonal Fuzzy Numbers”, International Journal of Mathematics And its Applications,Volume 4, Issue 4 (2016), 381-387. (Special Issue)

Authors Profile

CHARLES RABINSON G Assistant Professor, Department of Mathematics

VISTAS, Chennai. I have graduated from St. Joseph’s College, Trichirappalli in 2001 and Master degree and M.Phil degree from in same institution in 2004 and 2006 respectively. Now I am doing Ph.D in VISTAS, Chennai. My specialization in Optimization Techniques, Fuzzy Set and Fuzzy Logic.