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On the Edge Coloring of Triangular Snake Graph Families

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ABSTRACT

We discuss the edge chromatic number of the triangular snake graph T_n, double triangular snake graph DT_n, triple triangular snake graph TT_n and alternate triangular snake graph AT_n. A proper edge coloring of a graph G, is an assignment of colors to all the edges of graph G so that the adjacent edges received distinct colors. The smallest number of colors needed for such coloring is known as edge chromatic number.

Keywords: Triangular snake graph, double triangular snake graph, triple triangular snake graph, alternate triangular snake and edge coloring.

2010 Mathematics Subject Classification: 05C15.

1. INTRODUCTION

All graphs considered in this article are finite, simple and undirected. Let $G=(V(G),E(G))$ be a graph consists of a vertex set $V(G)$ and edge set $E(G)$ respectively. In 1880, Tait [4] was introduced the concept of edge coloring and he proved that, if the four-color conjecture is true then the edges of all the 3-connected planar graph can be colored properly only using 3-colors. In 1916, Konigsberg was proved that all the bipartite graphs have been edge colored with $\Delta(G)$ colors exactly. In 1949, Shannon[3] proved that all the graph have been edge colored with $\leq \frac{3}{2}\Delta(G)$ colors. In 1964, Vizing[5] proved that for every simple graph G, $\chi^{'}(G) \leq \Delta(G)+1$.

An edge coloring of a graph G is that an assignment of colors to the edges of G such that the adjacent edges received distinct colors. The chromatic index of a graph G, denoted by $\chi^{'}(G)$, is the minimum number of colors required for a proper edge coloring of graph G. The graph G is k-edge-chromatic if $\chi^{'}(G)=k$. Obviously $\chi^{'}(G) \geq \Delta(G)$, where $\Delta(G)$ is the maximum degree of a graph G.

In other words, An edge coloring of graph G is a function $c: E(G) \rightarrow \{1, 2, ..., \Delta\}$, the colors satisfying the following conditions.

(i) $c(e) \neq c(e^{'})$ for any two adjacent edges $e, e^{'} \in E(G)$

The minimum number of colors are required for such coloring is called edge chromatic number of G and it is denoted by $\chi^{'}(G)$.

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Many real life situations can be modeled as a graph coloring problem, some of them are planning and scheduling problems, timetabling and map coloring. Since graph coloring problem is a NP-hard problem, until now there are not known deterministic methods as a whole that can solve such problems.

2. PRELIMINARIES

Definition 2.1: A *Triangular snake graph*[2] T_n is obtained from a path $\{u_1, u_2, ..., u_n\}$ by joining u_k and u_{k+1} to a new vertex v_k for $k \in \{1, 2, ..., n\}$. That is, every edge of a path is replaced by a triangle.

Definition 2.2: The d*ouble triangular snake graph* [2] DT_n consists of two triangular snakes that have a common path.

Definition 2.3: The *triple triangular snake graph* [2] TT_n consists of three triangular snakes that have a common path.

Definition 2.4: An a*lternate triangular snake graph* [2] AT_n is obtained by a path $\{u_1, u_2, ..., u_n\}$ by joining u_k and u_{k+1} to a new vertex alternatively v_k for $k \in \{1, 3, 5.....\}$,i.e. Every alternate edge of a path is replaced by triangle.

In this paper, we focus on edge chromatic number for triangular snake graph T_n, double triangular snake graph DT_n, triple triangular snake graph TT_n and alternate triangular snake graph AT_n.

3. MAIN RESULTS

Theorem 3.1. Let T_n be the triangular snake graph of order $n \geq 3$, then $\chi'(T_n) = 4$.

Proof. Let $V(T_n) = \{u_l : 1 \leq l \leq n-1\} \cup \{v_l : 1 \leq l \leq n\}$ and

$E(T_n) = \{e_l : 1 \leq l \leq n-1\} \cup \{s_l : 1 \leq l \leq n-1\} \cup \{f_l : 1 \leq l \leq n-1\}$, where the edges $\{e_l : 1 \leq l \leq n-1\}$ represents the edge $\{v_l v_{l+1} : 1 \leq l \leq n-1\}$, the edges $\{s_l : 1 \leq l \leq n-1\}$ represents the edge $\{u_l v_l : 1 \leq l \leq n-1\}$ and the edges $\{f_l : 1 \leq l \leq n-1\}$ represents the edge $\{u_l v_{l+1} : 1 \leq l \leq n-1\}$

Define an edge coloring $c : E(T_n) \to \{1, 2, 3, ..., \Delta\}$ as follows. Now we assign the edge coloring to all the edges as follows,

$$c(v_l v_{l+1}) = \begin{cases} 1, \text{ if } l \text{ is odd} \\ 2, \text{ if } l \text{ is even} \end{cases}$$

$$c(v_l u_l) = 3, c(u_l v_{l+1}) = 4$$

We observed that the procedure of edge coloring pattern, the graph T_n is edge colored properly with 4 colors. This implies that $\chi'(T_n) \leq 4$. Since $\Delta = 4$ and $\chi'(T_n) \geq \Delta = 4$. Therefore $\chi'(T_n) = 4$. Thus *c* is edge colored with 3 colors.

Theorem 3.2. Let DT_n be double triangular snake graph of order $n \geq 3$, then $\chi'(DT_n) = \Delta(DT_n) = 6$.

Proof. Let $V(DT_n) = \{u_l, w_l : 1 \leq l \leq n-1\} \cup \{v_l : 1 \leq l \leq n\}$ and

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$E(DT_n)=\begin{cases}\{e_l:1\le l\le n-1\}\bigcup\{e_l^{'}:1\le l\le n-1\}\bigcup\\ \{e_l^{''}:1\le l\le n-1\}\bigcup\{s_l:1\le l\le n-1\}\bigcup\{s_l^{'}:1\le l\le n-1\}\end{cases}$, where the edges $\{e_l:1\le l\le n-1\}$ represents the edge $\{v_lv_{l+1}:1\le l\le n-1\}$, the edges $\{e_l^{'}:1\le l\le n-1\}$ represents the edge $\{u_lv_l:1\le l\le n-1\}$, the edges $\{e_l^{''}:1\le l\le n-1\}$ represents the edge $\{u_lv_{l+1}:1\le l\le n-1\}$, the edges $\{s_l:1\le l\le n-1\}$ represents the edge $\{w_lv_l:1\le l\le n-1\}$ and the edges $\{s_l^{'}:1\le l\le n-1\}$ represents the edge $\{w_lv_{l+1}:1\le l\le n-1\}$

Define an edge coloring $c:E(DT_n)\to\{1,2,3,....,\Delta\}$ as follows. Now we assign the edge coloring to all the edges as follows,

$$c(v_lv_{l+1})=\begin{cases}1, \text{ if } l \text{ is odd}\\ 2, \text{ if } l \text{ is even}\end{cases}$$

$c(v_lu_l)=3$, $\quad c(u_lv_{l+1})=4$

$c(v_lw_l)=5$, $\quad c(w_lv_{l+1})=6$

Clearly the above method of edge coloring, the graph DT_n is edge colored properly with 6 colors. This implies that $\chi^{'}(DT_n)\le 6$. Since $\Delta=6$ and $\chi^{'}(DT_n)\ge\Delta=6$. Therefore $\chi^{'}(DT_n)=6$. Thus *c* is edge colored with 6 colors.

Theorem 3.3. Let TT_n be triple triangular snake graph, then $\chi^{'}(TT_n)=\Delta(TT_n), n\ge 3.$

Proof. Let $V(TT_n)=\{u_l,s_l,w_l:1\le l\le n-1\}\bigcup\{v_l:1\le l\le n\}$ and

$E(TT_n)=\begin{cases}\{e_l:1\le l\le n-1\}\bigcup\{e_l^{'}:1\le l\le n-1\}\bigcup\\ \{e_l^{''}:1\le l\le n-1\}\bigcup\{e_l^{'''}:1\le l\le n-1\}\bigcup\\ \{f_l:1\le l\le n-1\}\bigcup\{f_l^{'}:1\le l\le n-1\}\bigcup\{f_l^{''}:1\le l\le n-1\}\end{cases}$, where the edges $\{e_l:1\le l\le n-1\}$ represents the edge $\{v_lv_{l+1}:1\le l\le n-1\}$, the edges $\{e_l^{'}:1\le l\le n-1\}$ represents the edge $\{u_lv_l:1\le l\le n-1\}$, the edges $\{e_l^{''}:1\le l\le n-1\}$ represents the edge $\{u_lv_{l+1}:1\le l\le n-1\}$, the edges $\{e_l^{'''}:1\le l\le n-1\}$ represents the edge $\{u_ls_l:1\le l\le n-1\}$, the edges $\{f_l:1\le l\le n-1\}$ represents the edge $\{w_lv_l:1\le l\le n-1\}$, the edges $\{f_l^{'}:1\le l\le n-1\}$ represents the edge $\{w_lv_{l+1}:1\le l\le n-1\}$ and the edges $\{f_l^{''}:1\le l\le n-1\}$ represents the edge $\{s_lv_{l+1}:1\le l\le n-1\}$

Define an edge coloring $c:E(TT_n)\to\{1,2,3,....,\Delta\}$ as follows. Now we assign the edge coloring to all the edges as follows,

$$c(v_lv_{l+1})=\begin{cases}1, \text{ if } l \text{ is odd}\\ 2, \text{ if } l \text{ is even}\end{cases}$$

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$c(v_l u_l) = 3$, $c(u_l v_{l+1}) = 4$

$c(v_l w_l) = 5$, $c(w_l v_{l+1}) = 6$

$c(v_l s_l) = 7$, $c(s_l v_{l+1}) = 8$

We observed that the above condition of edge coloring, the graph TT_n is properly edge colored with 8 colors. Hence $\chi^{'}(TT_n) \le \Delta = 8$. Since $\Delta = 8$ and $\chi^{'}(TT_n) \ge \Delta = 8$. Therefore $\chi^{'}(TT_n) = 8$. Thus *c* is edge colored with 8 colors.

Theorem 3.4. Let AT_n be the alternate triangular snake graph, then $\chi^{'}(AT_n) = \Delta(AT_n), n \ge 3.$

Proof. Let $V(AT_n) = \{u_l : l \in \{1,2,....,n\}\} \bigcup \{v_l : l \in \{1,3,5,....,n-2\}\}$ and

Let $E(TT_n) = \{e_l : l \in \{1,2,....,n-1\}\} \bigcup \{e_l^{'} : l \in \{1,3,....,n-2\}\} \bigcup \{e_l^{''} : l \in \{1,3,....,n-2\}\}$,

where the edges $\{e_l : l \in \{1,2,....,n\}\}$ represents the edge $\{u_l u_{l+1} : l \in \{1,2,....,n-1\}\}$, the edges $\{e_l^{'} : l \in \{1,3,....,n-2\}\}$ represents the edge $\{u_l v_l : l \in \{1,3,....,n-2\}\}$, the edges $\{e_l^{''} : l \in \{1,3,....,n-2\}\}$ represents the edge $\{v_l u_{l+1} : l \in \{1,3,....,n-2\}\}$,

Define an edge coloring $c : E(AT_n) \to \{1,2,3\}$ as follows. Now we assign the edge coloring to all the edges as follows. Consider the following two cases

Case (i): when *n* is odd,

Subcase(i): $n = 2k+1, k = 2,4,6,.......$

$$c(e_l) = \begin{cases} 1, & \text{if } l \in \{1,3,5,....n-2\} \\ 2, & \text{if } l = 2k-2, k \in \{2,4,6,....,n-3\} \\ 3, & \text{if } l = 2k+2, k \in \{1,3,5,....,n-1\} \end{cases}$$

For $k \in \{1,3,5,....,n-1\}$

$$c(e_l^{'}) = \begin{cases} 2, & \text{if } l = 2k-1, \\ 3, & \text{if } l = 2k+2, \end{cases}$$

$$c(e_l^{''}) = \begin{cases} 3, & \text{if } l = 2k-1, \\ 2, & \text{if } l = 2k+2, \end{cases}$$

Subcase(ii): $n = 2k+3, k = 2,4,6,.......$

$$c(e_l) = \begin{cases} 1, & \text{if } l \in \{1,3,5,....n-2\} \\ 2, & \text{if } l = 2k-2, k \in \{2,4,6,....,n-1\} \\ 3, & \text{if } l = 2k+2, k \in \{1,3,5,....,n-3\} \end{cases}$$

For $k \in \{1,3,5,....,n-2\}$

$$c(e_l^{'}) = \begin{cases} 2, & \text{if } l = 2k-1, \\ 3, & \text{if } l = 2k+2, \end{cases}$$

$$c(e_l^{''}) = \begin{cases} 3, & \text{if } l = 2k-1, \\ 2, & \text{if } l = 2k+2, \end{cases}$$

Case (i): when *n* is even,

$$c(e_l) = \begin{cases} 1, & \text{if } l \in \{1,3,5,....,n-1\} \\ 2, & \text{if } l = 2k-2, k \in \{2,4,6,....,n-4\} \\ 3, & \text{if } l = 2k+2, k \in \{1,3,5,....,n-2\} \end{cases}$$

For $k \in \{1,3,5,....,n-1\}$

$$c(e_l^{'}) = \begin{cases} 2, & \text{if } l = 2k-1, \\ 3, & \text{if } l = 2k+2, \end{cases}$$

$$c(e_l^{''}) = \begin{cases} 3, & \text{if } l = 2k-1, \\ 2, & \text{if } l = 2k+2, \end{cases}$$

We have observed that the above condition of edge coloring, the graph AT_n is properly edge colored with 3 colors. This implies that $\chi^{'}(T_n) \leq \Delta = 3$. Since $\Delta = 3$ and $\chi^{'}(T_n) \geq \Delta = 3$. Therefore $\chi^{'}(T_n) = 3$. Thus *c* is edge colored with 3 colors.

4. CONCLUSION

In this article, we obtained an edge chromatic number of triangular snake graph T_n, double triangular snake graph DT_n, triple triangular snake graph TT_n and alternate triangular snake graph AT_n.

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