JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1815-1820
https://publishoa.com
ISSN: 1309-3452

Mathematical Modelling of Hydrocephalus in Porous Medium with Oscillatory Flow

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Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

ABSTRACT

CSF is Newtonian incompressible fluid with viscosity at 37°C is 0.7-1 mPaWs. The abnormal CSF flow simulation is modelled using highly nonlinear partial differential equations and solved analytically using perturbation techniques. Numerical results are graphically portrayed and significance of the parameters are given by its impact on the result through which it is diffused by hydrocephalus. The effect of various physical parameters such as Womersley number, Darcy number were discussed with various plot using MATLAB.

Key words: Cerebrospinal fluid, Darcy number, Hydrocephalus, Oscillatory flow, Womersley number.

1. Introduction

Fluid dynamics has wide applications in medical field. Cerebrospinal fluid is one among the biofluid that has been handled by most of the mathematicians to predict the pathological disorder. Recently, there arise an interest in the biofluid dynamical studies of various characteristics flow under various conditions. In this paper we developed a mathematical model for hydrocephalus patient with respect to its flow momentum and pressure.

Bering et. al described the changes that occur in the formation and absorption of cerebrospinal fluid within the cerebral ventricles during the development of hydrocephalus. The effects of structural and hydraulic differences of white and grey brain matter, and the ependymal lining surrounding the ventricles with steady-state distribution of Edema seen in hydrocephalus are investigated by Kaczmarek et al. Pressure differences between lateral ventricles and Subarachnoid space, Differences between the predicted and observed CSF flow velocities in the prepontine area point towards complex brain-CSF interactions in pulsatile CSF flow was shown by Linninger et al(2007). Numerical solutions of mechanisms for hydrocephalus: complete blockage of the aqueduct , normal pressure hydrocephalus (NPH), mechanical effects in idiopathic intracranial hypertension was portrayed in IAN SOBEY et.al. (2006). Mathematical models of hydrocephalus were reviewed numerically in Tenti & Sivaloganathan et.al. Computational models of the CSF mean pressure and pressure amplitude was studied by Gholampour et.al.

The flow of CSF which results in the development of hydrocephalus for animal was analysed by Rekate et.al. Minor pathway hydrocephalus based on the evolution theory of CSF dynamics was given by Symss et.al. Concepts of CSF flow dynamics and the pathophysiology of hydrocephalus on time spatial spin labelling inversion pulse imaging of CSF dynamics was analysed by Yamada et.al. The distribution of periventricular Ederma in acute hydrocephalus of increased intraventricular pressure and ventricular geometry was given by Pickard et al.

Cerebrospinal fluid is a colourless, watery fluid that secretes from four ventricles of choroid plexuses. It protects brain and plays a key role in Central nervous system. It flows between pia mater and superior sub arachnoid space. For an adult, choroid plexuses secrete 400-600 ml of CSF per day. The CSF produced is then later reabsorbed as blood by arachnoid villi, that circulates all over the body. Hydrocephalus is a kind of excess secretion of Cerebrospinal fluid which can causes damage in brain. Hence the excess fluid diffuses CSF nature.

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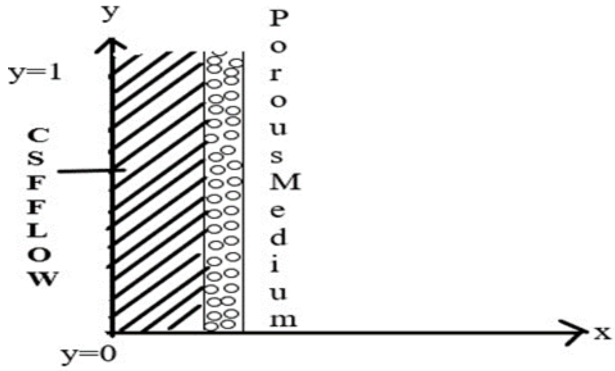

2. **Mathematical Formulation**

CSF is clearly water like incompressible viscous Newtonian fluid, we have considered an unsteady one-dimensional laminar flow of uniform cross section *h*, fluid is bounded by porous layer pia matter in subarachnoid space (SAS).

The spinal subarachnoid space SAS (filled with CSF) is a thin annular canal bounded internally by the pia mater, which surrounds the spinal cord, and externally by the rigid dura membrane. The following assumptions are made such as, the density of fluid is constant and the temperature between the particles is uniform throughout the fluid motion. The unsteady Darcy–Brinkman equation is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial P}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\frac{\nu}{k}u \quad (2)$$

the boundary conditions are assumed to be,

$$if\ y=0\ then\ u=0$$

$$if\ y=h\ then\quad \frac{\partial u}{\partial y}=\frac{\gamma}{\sqrt{k}}u$$

Fig:1 Physical Configuration

after non dimensionalisation the governing equations, we get

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0 \quad (3)$$

$$\alpha^2\left[\frac{\partial \bar{u}}{\partial \bar{t}}+\frac{\bar{u}(\partial \bar{u})}{\partial \bar{x}}+\frac{\bar{v}(\partial \bar{u})}{\partial \bar{y}}\right]=-\frac{\partial \bar{P}}{\partial \bar{x}}+\nu\left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right]-S^2\bar{u} \quad (4)$$

Were the boundary conditions being,

$$if\ \bar{y}=0\ \text{then}\ \bar{u}=0$$

$$if\ \bar{y}=h\ \text{then}\quad \frac{\partial \bar{u}}{\partial \bar{y}}=\gamma\sigma\bar{u}$$

Introducing the following dimensionless quantities,

$$u=\frac{\bar{u}}{\omega h};P=\frac{\bar{P}}{\rho\omega\nu};\ t=\omega\bar{t}\,;y=\frac{\bar{y}}{h}$$

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$\alpha^2 = \frac{\omega h^2}{\nu}$ (Womersley Number)

$Da = \frac{K}{h^2}$ (Darcy Number)

For our convenience we omit bar in the further results, we get

$$\alpha^2\left[\frac{\partial u}{\partial t} + \vartheta\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - S^2\bar{u} \quad (5)$$

$$if\ y = 0\ \ then\ u = 0$$

$$if\ y = 1\ then\quad \frac{\partial u}{\partial y} = \gamma\sigma u$$

3. Method of Solution

The above governing equations were solved analytically using perturbation method. This method can be done by representing perturbation parameter ($\epsilon\ << 1$) is very small flow velocity u as follows,

$$u(x, y, t) = u_0 + \ \epsilon\ e^{nt}u_1 + o(\epsilon^2) \quad (6)$$

$$P(x, y, t) = g + \ \epsilon\ e^{nt}P_1 + o(\epsilon^2) \quad (7)$$

Using equations (5) in (6) and (7)

$$\alpha^2\vartheta\frac{\partial u_0}{\partial y} = -g + \frac{\partial^2 u_0}{\partial y^2} - (S^2)u_0 \quad (8)$$

$$\alpha^2\left[nu_1 + \vartheta\frac{\partial u_1}{\partial y}\right] = \frac{\partial^2 u_1}{\partial y^2} - (S^2)u_1 \quad (9)$$

Solving the above resultant equations, we get the following base and perturb part of momentum equation.

$$u_0 = A_1e^{m_1 y} + A_2e^{m_2 y} - \frac{g}{a} \quad (10)$$

$$u_1 = A_3e^{m_3 y} + A_4e^{m_4 y} - \frac{1}{a + n\ \alpha^2} \quad (11)$$

$$u(x, y, t) = A_1e^{m_1 y} + A_2e^{m_2 y} - \frac{g}{a} +$$

$$\epsilon\ e^{nt}\left(A_3e^{m_3 y} + A_4e^{m_4 y} - \frac{1}{a + n\ \alpha^2}\right) \quad (12)$$

4. Result and discussion

we consider the flow as oscillatory and the excess fluid passes through the walls is porous medium. The increase of CSF which forms eddies in each ventricle that affects the velocity of flow in each field which is abnormal to human nature. In this paper we tried to expose those abnormalities by plotting graphically with few parameters like Darcy number, Wormsley number and so on.

Analytical solutions of this problem are performed and the results are portrayed graphically in Figs. 2–19 to show the interesting features of significant physical parameters on the velocity distributions. Throughout the computations we employ ($t = 0.9, \mu = 0.001003$,

$\rho = 998.2$ [3], $\lambda = 0.05, \epsilon = 0.01, n = 0.1, x = 0.01, \vartheta = 0.8, k = 3.3 \times 10^{-15}$[3], $\alpha = 0.4$

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As pressure increases velocity of CSF increases with respect to the length of tissue

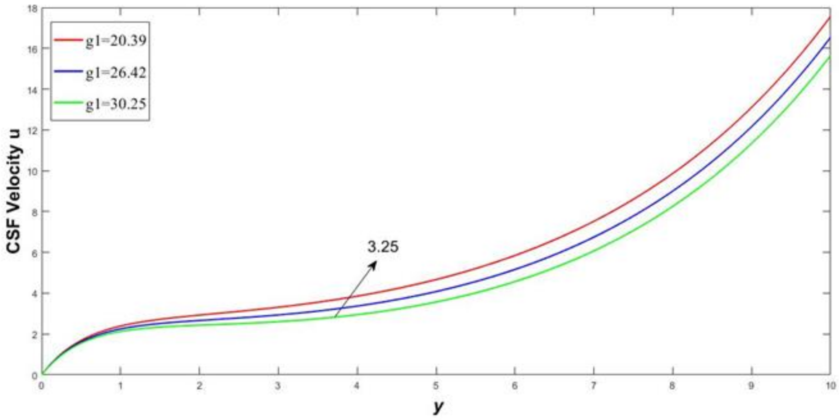

Fig: 3. Velocity for different values of $g1$

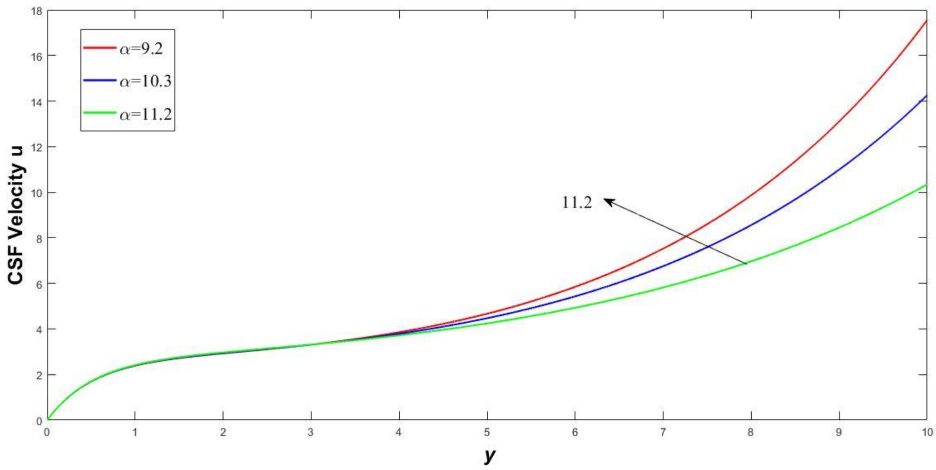

Fig: 3. Velocity for different values of α

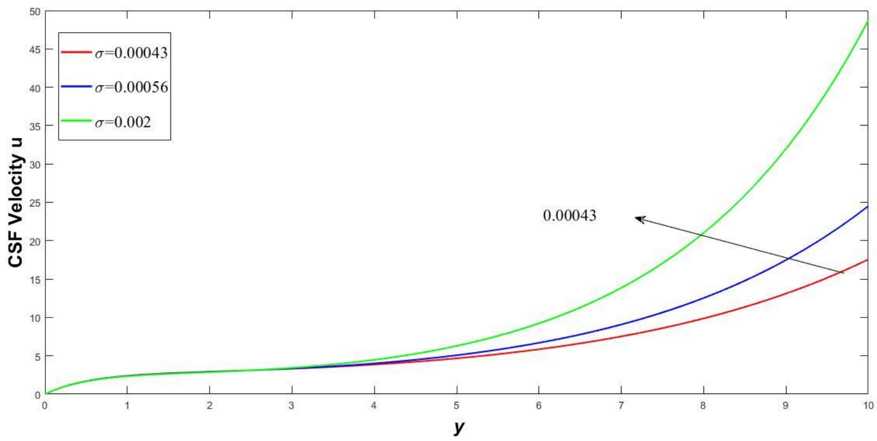

Fig:4 Velocity variation for different Darcy number

From the above graphs we summarized the velocity of fluid flow increases gradually for increasing Wormsley number, Darcy number and pressure difference. The flow Velocity of Porosity is increasing slowly with respect to characteristic length of cerebrospinal fluid.

5. Conclusions

Hydrocephalus can be diagonalised by various symptoms line MRI, Ultrasound, Radiography and so on. In this paper we tried to conclude its symptoms analytically and graphically. Future this can be extended for pandemic disease COVID - 19 as it was proved that the pandemic virus affects brain physiology

Appendix

$$m_1 = \frac{\alpha^2\vartheta + \sqrt{(\alpha^2\vartheta)^2 + 4a}}{2} \quad , m_2 = \frac{\alpha^2\vartheta - \sqrt{(\alpha^2\vartheta)^2 + 4a}}{2} \quad , a = S^2$$

$$A_1 = \frac{g}{a} - A_2 \quad , A_2 = \frac{\frac{g}{a}[\gamma + e^{m_1}(m_1 - 1)]}{e^{m_2}(\gamma - m_2) + e^{m_1}(m_1 - \gamma)}, \; S^2 = \frac{1}{Da}$$

$$A_2 = \frac{\frac{g}{a}[\gamma\sigma + e^{m_1}(m_1 - 1)]}{e^{m_2}(\gamma\sigma - m_2) + e^{m_1}(m_1 - \gamma\sigma)}, A_3 = \frac{1}{a + n\,\alpha^2} - A_4$$

$$A_4 = \frac{\frac{1}{a + n\,\alpha^2}[\gamma\sigma + e^{m_3}(m_3 - \gamma\sigma)]}{e^{m_4}(\gamma\sigma - m_4) + e^{m_3}(m_3 - \gamma\sigma)}$$

$$m_5 = \frac{l + \sqrt{l^2 + 4n}}{2} \; ; \; m_6 = \frac{l - \sqrt{l^2 + 4n}}{2}.$$

Nomenclature

CSF → Cerebrospinal Fluid

u, v → velocity of the fluid flow in x and y direction.

ρ → density of the fluid

P → pressure (Systolic and Diastolic)

k → permeability of the porous layer

ν → kinematic viscosity of CSF

References

[1] M. Keith Sharp et al, "Dispersion in porous media in oscillatory flow between fat plates: applications to intrathecal, periarterial and para arterial solute transport in the central nervous system", *Fluids Barriers CNS (2019),* https://doi.org/10.1186/s12987-019-0132-y.

[2] A. L. Sánchez et al, "On the bulk motion of the cerebrospinal fluid in the spinal canal", *J. Fluid Mech. (2018), vol. 841*, https://doi.org/10.1017/jfm.2018.67

[3] Andreas A. Lininger et al, "Cerebrospinal Fluid Flow in the Normal and Hydrocephalic Human Brain", *IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 54, NO. 2, (2007),* https://doi.org/10.1109/TBME.2006.886853.

[4] Jordi Alastruey et al, "Arterial pulse wave haemodynamic", *In S. Anderson (Ed.), 11th International Conference on Pressure Surges. (2012). (pp. 401-443). Lisbon: Virtual PiE Led t/a BHR Group*.

[5] S. Kalyanasundaram et al, "A finite element model for predicting the distribution of drugs delivered intracranially to the brain", *Am. J. Physiol. 273 (Regulatory Integrative Comp. Physiol. 42): R1810– R1821, (1997).* *https://doi.org/10.1152/ajpregu.1997.273.5.r1810*.

[6] Edgar A. Bering, JR., M.D. et al, "Hydrocephalus: Changes in Formation and Absorption of Cerebrospinal Fluid Within the Cerebral Ventricles", *Journal of neurosurgeon (1963), Vol 20-Issue 12* *https://doi.org/10.3171/jns.1963.20.12.1050*.

[7] Mauro Ursino, "A Mathematical study of human intracranial hydrodynamics part I-The cerebrospinal fluid pulse pressure", *Annals of Biomedical Engineering, Vol. 16, pp. 379-401, (1988), https://doi.org/10.1007/BF02364625*.

[8] Nigel Peter Syms et al , "Theories of cerebrospinal fluid dynamics and hydrocephalus: historical trend: A Review ", *J Neurosurg Pediatrics 11:170–177, (2013), https://doi.org/10.3171/2012.3.PEDS0934*.

[9] Kauffman J., Drapaca C.S. (2014) "A Fractional Pressure-Volume Model of Cerebrospinal Fluid Dynamics in Hydrocephalus. In:" Barthelat F., Zavattieri P., Korach C., Prorok B., Grande-Allen K. (eds) *Mechanics of Biological Systems and Materials, Volume 4*. Springer, Cham. *https://doi..org/10.1007/978-3-319-00777-9_26*.

[10] G. Tenti, J.M. Drake & S. Sivaloganathan, " Brain biomechanics: Mathematical modeling of hydrocephalus", *Neurological Research, 22:1, 19-24, DOI: 10.1080/01616412.2000.11741033(2000)*.

[11] S. Gholampour et.al," Numerical simulation of Cerebrospinal Fluid hydrodynamics in the healing process of hydrocephalus patients", *ISSN 0021-8944, Journal of Applied Mechanics and Technical Physics, 2017, Vol. 58, No. 3, pp. 386–391, DOI: 10.1134/S0021894417030026*