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An EOQ Model with Price Dependent Demand and Partially Backlogging Shortages with Salvage Cost

M. Sathishkumar[1], T. Iswarya[2]

[1&2] Department of Mathematics
Vels Institute of Science, Technology & Advanced Studies, Chennai, India
Mail ID: Iswarya.sbs@velsuniv.ac.in

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Abstract:

This paper deal with deterministic inventory model for deteriorating item with shortages that are partially backlogging and salvage cost is associated to the deteriorating items. This model discusses about the demand rate which is price dependent with the negative power of constant. The holding cost and the deterioration cost is both time dependent. To represent the solution and application of the model, numerical analysis has been provided. By finding the minimum value for the total cost, the model is solved. To represent the model well a sensitivity analysis is discussed for the effect of changes of the parameters.

Keywords: Price dependent demand, Weibull distribution, partially backlogged shortages, deterioration cost, salvage value, linear holding cost.

1. Introduction

Inventory management primarily deals with stocking and storing of produced goods. Inventory management is required at many locations like facility or supply network so that plans can be made for producing, stocking and storing of materials.

The scope of inventory management deals with lead time, holding cost, production cost, cost of carrying the stocks, asset management, predicting, replenishment, deteriorating and defective goods, returns, quality and quantity management, demand and shortage forecasting, maintaining the physical inventory and physical space for the inventory that is kept in the available space.

The papers of Pareek & Kumar [12], Mishra & Singh [11] are taken and made more realistic by adding the salvage value that is associated with the deteriorating items. Both the deterioration rate and the holding cost that is used in this model is time dependent. Shortages are allowed and included with salvage cost. Shortages are partially backlogging with Weibull distribution. All the cost used in the model are optimized which gives the minimal value for the Total Cost

Assumptions

The following assumptions are made to establish the model.

1. Demand is price dependent.
2. The lead time is zero or negligible.
3. The replenishment rate is infinite.
4. Shortages are partially backlogged and is given by

$$\prod(t) = \frac{1}{1+\delta(T-t)}$$

5. Deterioration is time dependent and it is defined by weibull's distribution i.e., $\theta(t) = \alpha\beta t^{\beta-1}$ where α ($0 \le \alpha \le 1$) and $\beta > 1$.
6. The salvage value γ ($0 \le \gamma \le 1$) is associated with the deteriorating items during the cycle time.

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7. The holding cost is a time dependent and it is given by a linear function. i.e., H(t) = a + bt (a>0, b>0).

2. MODEL FORMULATION:

The changes of the inventory level $[0, t_1]$ and shortage period $[t_1, T]$ is given by the following differential equations.

$$\frac{dI_1(\mathrm{t})}{dt} + \theta(\mathrm{t})I_1(\mathrm{t}) = -ap^{-\alpha}$$
$$\frac{dI_2(\mathrm{t})}{dt} = -ap^{-\alpha}$$

The boundary conditions are

$$I_1(\mathrm{t}) = I_2(\mathrm{t}) = 0 \text{ at } \mathrm{t} = t_1,$$

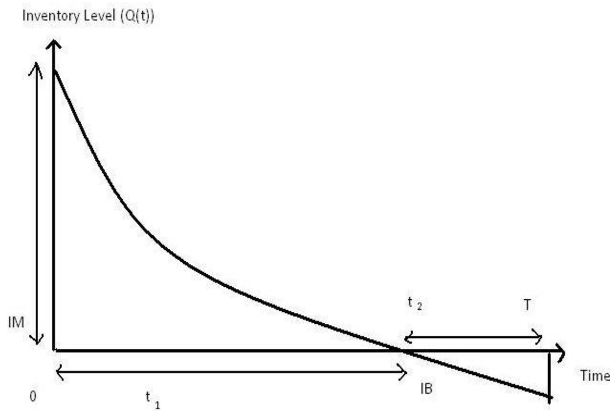
$$I_1(\mathrm{t}) = \mathrm{IM} \text{ and } I_2(\mathrm{t}) = \mathrm{IB} \text{ at } \mathrm{t} = \mathrm{T}$$

3. THEORETICAL SOLUTION OF THE MODEL

3.1. CASE I: INVENTORY LEVEL WITHOUT SHORTAGE

During the period $[0,t_1]$, the inventory depletes due to the deterioration and demand. Hence, the inventory level at any time during $[0,t_1]$ is described by differential equation

$$\frac{dI_1(\mathrm{t})}{dt} + \theta(\mathrm{t})I_1(\mathrm{t}) = -ap^{-\alpha}$$



With the boundary conditions are $I_1(0) = \mathrm{IM}$ and $I_1(t_1) = 0$.

$$I_1(t)e^{\alpha t^{\beta}} = -\int_{t_1}^{t} ap^{-\alpha}e^{\alpha t^{\beta}}dt$$

Ignoring the higher powers of α^2 and more

$$I_1(t) = ap^{-\alpha}\left[t_1 - t + \alpha t^{\beta+1}\,\frac{e^{\alpha {t_1}^{\beta+1}}}{\beta+1} - \frac{e^{\alpha t^{\beta+1}}}{\beta+1}\right]$$

5.2 CASE II: INVENTORY LEVEL WITH SHORTAGE

Inventory level during shortage period $[t_1, T]$ which is partially backlogged depends on the demand. The inventory during $[t_1, T]$ at this stage can be represented by a differential equation as follows.

$$\frac{dI_1(\mathrm{t})}{dt} = -ap^{-\alpha} \qquad \text{where } (t_1<\mathrm{t}<\mathrm{T})$$

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The boundary conditions are given as $I_2(t_1)$ =0 and I_2(t) = IB

$$\int dI_2(\mathrm{t}) = \int_{t_1}^{t} -ap^{-\alpha}\,dt$$

$$I_2(t) = -ap^{-\alpha}\int_{t_1}^{t} dt$$

$$I_2(t) = -ap^{-\alpha}(t - t_1)$$

The solution of differential equation is

$$I_2(t) = ap^{-\alpha}(t_1 - t)$$

Therefore, the total cost function has the following components:

5.3 Inventory Holding cost:

$$HC = \int_0^{t_1} H(t)I_1(\mathrm{t})\mathrm{dt}$$

$$= \int_0^{t_1} ap^{-\alpha}\left[t_1 - t + \alpha t^{\beta+1}\ \frac{e^{\alpha t_1{}^{\beta+1}}}{\beta+1} - \frac{e^{\alpha t^{\beta+1}}}{\beta+1}\right]\ (\mathrm{a + bt})\ \mathrm{dt}$$

$$IHC = a^2p^{-\alpha}\left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1{}^{\beta+2}}{(\beta+1)(\beta+2)}\right] + abp^{-\alpha}\left[\frac{t_1^3}{6} + \frac{\alpha\beta t_1{}^{\beta+3}}{2(\beta+2)(\beta+3)}\right]$$

5.4 **Shortage Cost during** $[t_1, T]$:

$$SC = \int_{t_1}^{T} ap^{-\alpha}\frac{1}{1+\delta(T-t)}\ (t_1 - t)dt$$

$$SC = ap^{-\alpha}\left[t_1T + \frac{(1+\delta T)t_1^2}{2} - \frac{(1+\delta t_1)T^2}{2} + \frac{\delta}{6}(t_1^3 + T^3)\right]$$

5.5 Loss of Stock due to deterioration

= Total inventory – demand

$$D = [I_1(0) - I_2(0)]$$

$$I_1(0) = \left[t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right]; I_2(0) = ap^{-\alpha}t_1$$

$$D = ap^{-\alpha}\left(\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)$$

5.6 Deterioration Cost:

$$CD = C[I_1(0) - I_2(0)]$$

$$CD = CR\left(\frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)$$

5.7 Ordering Cost per order:

OC = A

5.8 Salvage Value of deteriorating items per unit

$$SV = \gamma Cap^{-\alpha}[I_1(0) - I_2(0)]$$

$$SV = \gamma C a p^{-\alpha}\left[t_1 + \frac{\alpha {t_1}^{\beta+1}}{\beta+1} - t_1\right]$$

$$SV = \gamma C a p^{-\alpha}\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right]$$

5.9 Total cost per time unit:

$$TC(t_1, T) = \frac{1}{T}[HC + SC + OC + CD - SV]$$

$$TC(t_1,T) = \frac{1}{T}\left\{\begin{array}{c} a^2p^{-\alpha}\left[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + bap^{-\alpha}\left[\frac{t_1^3}{6} + \frac{\alpha\beta {t_1}^{\beta+3}}{2(\beta+2)(\beta+3)}\right] \\ + \left[t_1 T + \frac{(1+\delta T)t_1^2}{2} - \frac{(1+\delta t_1)T^2}{2} + \frac{\delta}{6}(t_1^3 + T^3)\right] \\ + ap^{-\alpha}\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] + Cap^{-\alpha}\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] + A + \gamma Cap^{-\alpha}\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] \end{array}\right\}$$

The Necessary condition for the total cost per time unit to be minimized is

$$\frac{\partial TC}{\partial t_1} = 0 \; and \; \frac{\partial TC}{\partial T} = 0$$

Provided

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0 \; and \; \left(\frac{\partial^2 TC}{\partial t_1^2}\right) > 0$$

$$\frac{\partial TC}{\partial t_1} = \frac{ap^{-\alpha}}{T}\left\{\begin{array}{l} a\left(t_1 + \frac{\alpha\beta {t_1}^{\beta+1}}{\beta+1}\right) + b\left(\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right) \\ +T + (1+\delta T)t_1 - \frac{\delta T^2}{2} + \frac{t_1^2}{2} + C\left(\alpha {t_1}^{\beta}\right) \end{array}\right\}$$

$$\frac{\partial TC}{\partial T} = \left\{\begin{array}{c} ap^{-\alpha}\left[\begin{array}{c} -\frac{a}{T^2}\left(\frac{t_1^2}{2} + \frac{\alpha\beta {t_1}^{\beta+1}}{(\beta+1)(\beta+2)}\right) \\ -\frac{b}{T^2}\left(\frac{t_1^3}{6} + \frac{\alpha\beta {t_1}^{\beta+3}}{2(\beta+2)(\beta+3)}\right) \\ -\frac{t_1^2}{2T^2} - \left(\frac{1}{2} + \frac{\delta t_1}{2}\right) - \frac{\delta t_1^3}{6T^2} - \frac{2\delta T}{6} \end{array}\right] \\ -\frac{A}{T^2} - \frac{(1-\gamma)CR}{T^2}\left[\frac{\alpha {t_1}^{\beta+1}}{\beta+1}\right] \end{array}\right\}$$

$$\frac{\partial^2 TC}{\partial t_1^2} = \frac{ap^{-\alpha}}{T}\left\{\begin{array}{c} a\left(1 + \alpha\beta t_1^{\beta}\right) + b\left(t_1 + \frac{\alpha\beta t_1^{\beta+1}}{2}\right) \\ +(1+\delta T) + t_1 + c\alpha\beta t_1^{\beta-1} \end{array}\right\}$$

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$$\frac{\partial^2 TC}{\partial T^2} = \left\{ \begin{array}{c} ap^{-\alpha}\left[\begin{array}{c} \frac{2a}{T^3}\left(\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right) \\ + \frac{2b}{T^3}\left(\frac{t_1^3}{6} + \frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right) \\ + \frac{t_1^2}{T^3} + \frac{2\delta t_1}{6T^3} - \frac{2\delta}{6} \end{array} \right] \\ + \frac{2A}{T^3} + \frac{2(1-\gamma)Cap^{-\alpha}}{T^3}\left[\frac{\alpha t_1^{\beta+1}}{\beta+1}\right] \end{array} \right\}$$

Since the nature of the total cost function is highly nonlinear thus the convexity of the function shown graphically in the next section.

4. Numerical Example and Sensitivity Analysis

Consider an inventory system with the following parametric values in proper units:

R= $ap^{-\alpha}$ =2000, , α=0.15, a=2.0, b=1.5, γ=0.10, C=10, A=50β=1.5, δ=0.20, The output of the program by using Mathematica software is T=1.4232174, t1=0.0188969 and TC= 70.30458743.

Substituting the above values in the above equations we get the values

Table: The Changes in the parameter which has the effect on the Inventory Model

Parameter	%Change	T	t_1	TC
R	+40%	1.419974154	0.011670694	70.41517307
	+20%	1.421459572	0.014699531	70.33800198
	-20%	1.424000823	0.019892019	70.20544019
	-40%	1.427021838	0.026078628	70.04716037
α	+40%	1.421875010	0.015634987	70.31200097
	+20%	1.422006364	0.015861027	70.30726672
	-20%	1.422288647	0.016343422	70.29726112
	-40%	1.422440703	0.016601406	70.29196482
β	+40%	1.422858829	0.017321357	70.27687360
	+20%	1.422670439	0.017019364	70.28255367
	-20%	1.42089432	0.013621193	70.36365588
	-40%	1.418544039	0.008423916	70.50600602
γ	+40%	1.422175498	0.016150631	70.30124324
	+20%	1.422159739	0.016123708	70.30180146
	-20%	1.422128476	0.016070253	70.30291106
	-40%	1.422112985	0.016043720	70.30346244
δ	+40%	1.420114480	0.011907428	70.44910654
	+20%	1.420985302	0.013696951	70.37548716
	-20%	1.423751974	0.019467654	70.22971250
	-40%	1.426105065	0.024501662	70.15754766
a	+40%	1.422121960	0.016063116	70.30295456
	+20%	1.422132988	0.016079981	70.30265332
	-20%	1.422155195	0.016113921	70.30206670
	-40%	1.422166378	0.016130997	70.30178114
	+40%	1.422176142	0.016132224	70.30187113

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b	+20%	1.422178114	0.016132241	70.30188126
	-20%	1.422188145	0.016144122	70.30188225
	-40%	1.422189963	0.016442144	70.30198220
p	+40%	1.422199232	0.016443217	70.30199236
	+20%	1.422199898	0.016443418	70.30199359
	-20%	1.422199909	0.016443527	70.30199445
	-40%	1.422201513	0.016443558	70.30199589

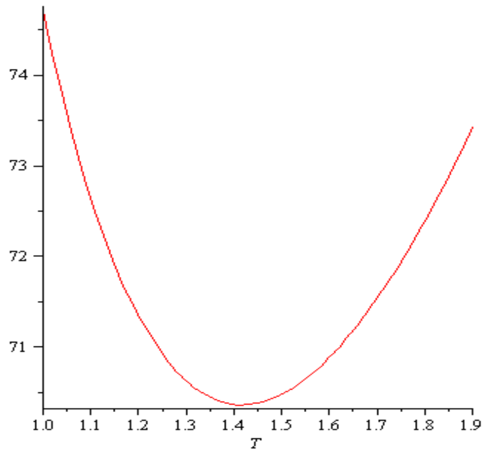

Figure-1 (T vs. TC at t_1=0.01)

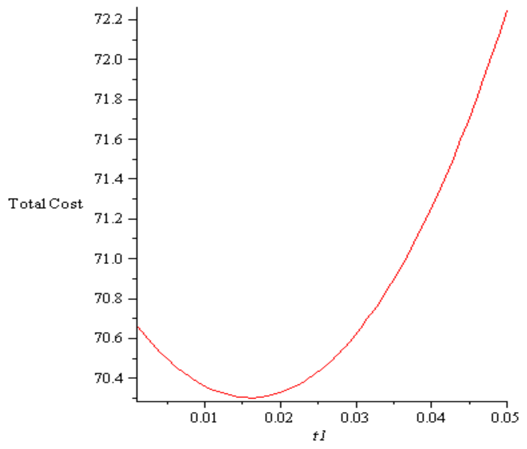

Figure-2 (t_1 vs T at T=1.42)

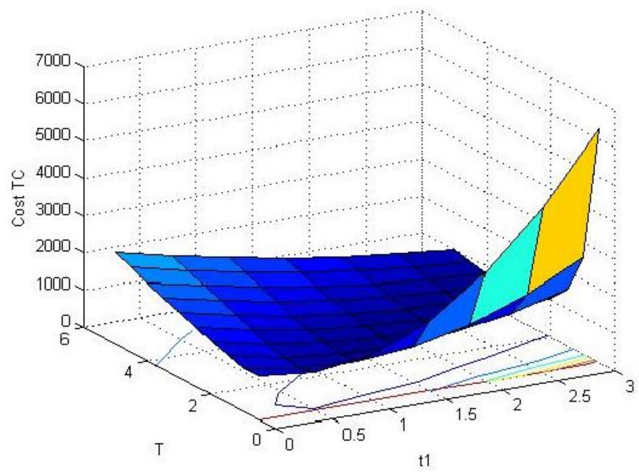

Figure – 3 (T vs t_1 vs T Vs TC)

5. Conclusion

In this paper, the demand considered is negative power of price. The Taylor series expansion is used to optimize the problem. A sensitivity analysis is carried out to illustrate and validate the effect of changes in one parameter while keeping the others fixed. To show that the total cost is an objective function which is convex, we supply three parameters of the objective function. From the above result we can conclude that the above model gives the analytical solution for minimal total cost. This model can be further developed into fuzzy and interval models.

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