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A Study On Strong Interval Valued Picture Fuzzy Graphs

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Abstract. In this paper, we introduced the strong interval valued picture fuzzy graph. The operations on the Cartesian product, Composition, Union and Join of two strong interval valued picture fuzzy graphs are defined. Some propositions are involving strong interval valued picture fuzzy graphs are stated and proved with suitable examples.

Keywords: Picture fuzzy graph, interval valued picture fuzzy graph, Strong interval picture fuzzy graph,

I Introduction

In 1965, Zadeh introduced the fuzzy set as the uncertainty of finding method. Fuzzy graph theory for finding more applications in real time problems with different levels of precision. The applied field of a fuzzy set theory depends on the membership function—the interval valued picture fuzzy set as a generalization of a picture fuzzy set. However, the calculation and expression are slightly different. The characteristic of the picture fuzzy model is that the sum of positive, neutral and negative membership degrees is not more significant than one. The picture fuzzy sets have higher potential than the intuitionistic fuzzy sets and fuzzy sets to manage the uncertainty. We develop systematic operators to aggregate the strong interval valued picture fuzzy graph (SIVPFG). Ahmed Mostafa Khalil and Sheng Gang Li [1] have introduced the new operations set on interval valued fuzzy picture fuzzy sets and soft sets. Krassimir Atanassov [4] has presented the four types of interval valued intuitionistic fuzzy graphs. Mishra and Pal [5] developed the regular interval valued intuitionistic fuzzy graphs. Said and Mohamed Talea [14] introduced strong interval valued Neutrosophic graphs and studied the edge interval valued fuzzy graph by Ravi Narayanan and Santhi [13].

Further, in [16], essential operations on interval valued Pythagorean fuzzy graphs are defined, and their properties are studied. In this paper, we introduced the strong interval valued picture fuzzy graph. Some operations like the Cartesian product, Composition, Union and Join of two strong interval valued picture fuzzy graphs are defined. Also, we discussed some propositions with suitable illustrations.

II Preliminaries

Definition:2.1 [intuitionistic fuzzy graph] Let X be a universe of discourse, An intuitionistic fuzzy set A in X is given by A = $\{< \mu_A(x), \vartheta_A(x) >/x \in X\}$, where $\mu_A: X \to [0,1]$ denotes the degree of membership and $\vartheta_A: X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A, respectively. With the condition that $0 \le \mu_A(x) + \vartheta_A(x) \le 1$.The degree of indeterminacy $\pi_A(x) = \sqrt{1 - \mu_A(x) - \vartheta_A(x)}$

Definition:2.2 [Strong intuitionistic fuzzy graph] An intuitionistic fuzzy graph $G = (A, B)$ is called a strong intuitionistic fuzzy graph if $\mu_B(x,y) = min(\mu_A(x), \mu_A(y))$ and $\vartheta_B(x,y) = max(\vartheta_A(x), \vartheta_A(y)), \forall (x,y) \in E$.

Definition:2.3 [interval valued fuzzy graph] The IVFG $G^* = (V, E)$, we mean that a pair $G = (A, B)$, where $A = [\mu_A^-(x), \mu_A^+(x)]$ is an IVFG on V and $B = [\mu_B^-(x), \mu_B^+(x)]$ is an IVFG on E, V is a non-empty set and $E \subseteq V \times V$.

Definition:2.4 [Join of two IVFGs] The join $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ of two IVFG $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ on ${G_1}^* = (V_1, E_1)$ and ${G_2}^* = (V_2, E_2)$ is defined as follows, $\{(\mu_{A_1}^- + \mu_{A_2}^-)(x) = (\mu_{A_1}^- \cup \mu_{A_2}^-)(x)$; $(\mu_{A_1}^+ + \mu_{A_2}^+)(x) = (\mu_{A_1}^+ \cup \mu_{A_2}^+)(x)\}, if\ x \in V_1 \cup V_2$, $\{(\mu_{B_1}^- + \mu_{B_2}^-)(xy) = (\mu_{B_1}^- \cup \mu_{B_2}^-)(xy)$; $(\mu_{B_1}^+ + \mu_{B_2}^+)(xy) = (\mu_{B_1}^+ \cup \mu_{B_2}^+)(xy)\}, if\ xy \in E_1 \cap E_2$, and $\{(\mu_{B_1}^- + \mu_{B_2}^-)(xy) = min(\mu_{A_1}^-(x), \mu_{A_2}^-(y))$; $(\mu_{B_1}^+ + \mu_{B_2}^+)(xy) = min(\mu_{A_1}^+(x), \mu_{A_2}^+(y))\}, if\ xy \in E'$, where E' is the set of all edges the vertex $V_1 and\ V_2$.

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III Strong Interval Valued Picture Fuzzy Graphs

Definition:3.1 An interval valued picture fuzzy graph (IVPFG) with underlying set V is defined to be a pair $G = (A, B)$ where A $= \{(\mu_{AL}, \eta_{AL}, \gamma_{AL};\ \mu_{AU}, \eta_{AU}, \gamma_{AU})\}$ is an interval valued picture fuzzy set on V and $B = \{(\mu_{BL}, \eta_{BL}, \gamma_{BL};\ \mu_{BU}, \eta_{BU}, \gamma_{BU})\}$ is an interval valued picture fuzzy set on E satisfies the following conditions: (i) the function $\mu_A: V \to [0,1]$; $\eta_A: V \to [0,1]$ and $\gamma_A: V \to [0,1]$ indicates positive membership degree, neutral membership degree and negative membership degree of the component $x \in V$, respectively, such that $0 \le \mu_A(x) + \eta_A(x) + \gamma_A(x) \le 1$ for all $x \in V$ and (ii) the function $\mu_B: V \times \mathrm{V} \to [0,1]$; $\eta_B: V \times \mathrm{V} \to [0,1]$ and $\gamma_B: V \times \mathrm{V} \to [0,1]$ is defined by

$\mu_{BL}(uv) \le \min(\mu_{AL}(u), \mu_{AL}(v))$; $\eta_{BL}(uv) \le \min(\eta_{AL}(u), \eta_{AL}(v))$ and $\gamma_{BL}(uv) \ge \max(\gamma_{AL}(u), \gamma_{AL}(v))$ and $\mu_{BU}(uv) \le \min(\mu_{AU}(u), \mu_{AU}(v))$; $\eta_{BU}(uv) \le \min(\eta_{AU}(u), \eta_{AU}(v))$ and $\gamma_{BU}(uv) \ge \max(\gamma_{AU}(u), \gamma_{AU}(v))$ such that $0 \le \mu_{BU}(uv) + \eta_{BU}(uv) + \gamma_{BU}(uv) \le 1$ for all uv$\in E$.

Example:3.1 Consider $G = (A, B)$ defined on a graph $G^* = (V, E)$ such that the vertex set V = {x, y, z} and the edge set E = {xy, yz, zx}. Let A be an interval valued picture fuzzy set of V and let B be interval valued picture fuzzy set of $E \subseteq V \times V$.

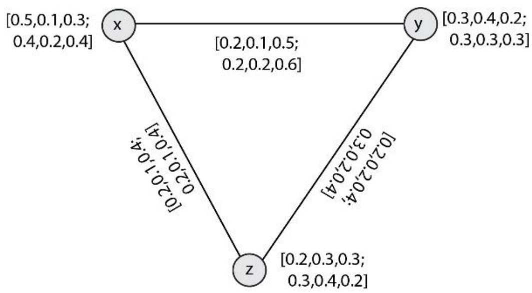

Fig 1: Interval valued picture fuzzy graph

Here A = {<x; [0.5, 0.1, 0.3], [0.4, 0.2, 0.4]>, <y; [0.3, 0.4, 0.2], [0.3, 0.3, 0.3]>, <z; [0.2, 0.3, 0.3],[0.3, 0.4, 0.2]>}, and B = {<xy; [0.2, 0.1, 0.5], [02, 0.2, 0.6]>, <yz, [0.2, 0.2, 0.4], [0.3, 0.2, 0.4]>, <zx; [0.2, 0.1, 0.4], [0.2, 0.1, 0.4]>}.

Definition:3.2 An interval valued picture fuzzy graph $G = (A, B)$ is called strong interval valued picture fuzzy graph (SIVPFG) if $\mu_{BL}(uv) = \min(\mu_{AL}(u), \mu_{AL}(v))$; $\eta_{BL}(uv) = \min(\eta_{AL}(u), \eta_{AL}(v))$ and $\gamma_{BL}(uv) = \max(\gamma_{AL}(u), \gamma_{AL}(v))$ and $\mu_{BU}(uv) = \min(\mu_{AU}(u), \mu_{AU}(v))$; $\eta_{BU}(uv) = \min(\eta_{AU}(u), \eta_{AU}(v))$ and $\gamma_{BU}(uv) = \max(\gamma_{AU}(u), \gamma_{AU}(v))$ such that $0 \le \mu_{AU}(uv) + \eta_{AU}(uv) + \gamma_{AU}(uv) \le 1$ for all uv$\in E$.

Example:3.2 Consider $G = (A, B)$ defined on a graph $G^* = (V, E)$ such that the vertex set V = $\{x_1, x_2, x_3, x_4\}$ and the edge set E = $\{x_1x_2, x_2x_3, x_3x_4, x_4x_1\}$. Let A be an interval valued picture fuzzy set of V and let B be interval valued picture fuzzy set of $E \subseteq V \times V$.

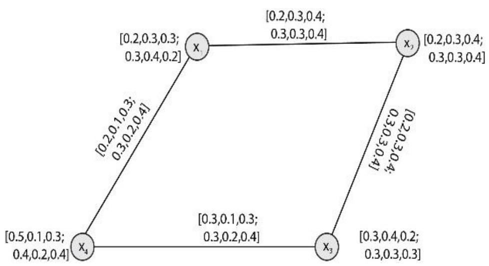

Fig 2: Strong Interval valued picture fuzzy graph

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Here A = {<x_1; [0.2, 0.3, 0.3], [0.3,0.4, 0.2]>, <x_2; [0.2, 0.3, 0.4], [0.3, 0.3, 0.4]>, <x_3; [0.3,0.4, 0.2],[0.3, 0.3, 0.3]>, <x_4; [0.5, 0.1, 0.3], [0.4, 0.2, 0.4]} and B = {<x_1x_2; [0.2, 0.3, 0.4], [0.3, 0.3, 0.4]>, <x_2x_3, [0.2, 0.3, 0.4], [0.3, 0.3, 0.4]>, <x_3x_4; [0.3,0.1, 0.3], [0.3, 0.2, 0.4]>, <x_4x_1; [0.2, 0.1, 0.3], [0.3, 0.2, 0.4]>

Definition:3.3 Two strong interval valued picture fuzzy graphs G_1 and G_2 of the underlying crisp graphs $G_1{}^*$and $G_2{}^*$. The cartesian product of two IVPFS G_1 and G_2 is denoted by

$G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ and defined as follows

(i) $(\mu_{\mathcal{A}_1L} \times \mu_{\mathcal{A}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{A}_1L}(\mathrm{u}_1), \mu_{\mathcal{A}_2L}(\mathrm{u}_2)\}$

$(\mu_{\mathcal{A}_1U} \times \mu_{\mathcal{A}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{A}_1U}(\mathrm{u}_1), \mu_{\mathcal{A}_2U}(\mathrm{u}_2)\}$

$(\eta_{\mathcal{A}_1L} \times \eta_{\mathcal{A}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{A}_1L}(\mathrm{u}_1), \eta_{\mathcal{A}_2L}(\mathrm{u}_2)\}$

$(\eta_{\mathcal{A}_1U} \times \eta_{\mathcal{A}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{A}_1U}(\mathrm{u}_1), \eta_{\mathcal{A}_2U}(\mathrm{u}_2)\}$

$(\gamma_{\mathcal{A}_1L} \times \gamma_{\mathcal{A}_2L})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{A}_1L}(\mathrm{u}_1), \gamma_{\mathcal{A}_2L}(\mathrm{u}_2)\}$

$(\gamma_{\mathcal{A}_1U} \times \gamma_{\mathcal{A}_2U})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{A}_1U}(\mathrm{u}_1), \gamma_{\mathcal{A}_2U}(\mathrm{u}_2)\}$, for every $\mathrm{u}_1 \in V_1$ & $\mathrm{u}_2 \in \mathrm{V}_2$

(ii) $(\mu_{\mathcal{B}_1L} \times \mu_{\mathcal{B}_2L})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{A}_1L}(\mathrm{u}), \mu_{\mathcal{B}_2L}(\mathrm{u}_2, v_2)\}$

$(\mu_{\mathcal{B}_1U} \times \mu_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{A}_1U}(\mathrm{u}), \mu_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$

$(\eta_{\mathcal{B}_1L} \times \eta_{\mathcal{B}_2L})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{A}_1L}(\mathrm{u}), \eta_{\mathcal{B}_2L}(\mathrm{u}_2, v_2)\}$

$(\eta_{\mathcal{B}_1U} \times \eta_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{A}_1U}(\mathrm{u}), \eta_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$

$(\gamma_{\mathcal{B}_1L} \times \gamma_{\mathcal{B}_2L})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{A}_1L}(\mathrm{u}), \gamma_{\mathcal{B}_2L}(\mathrm{u}_2, v_2)\}$

$(\gamma_{\mathcal{B}_1U} \times \gamma_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{A}_1U}(\mathrm{u}), \gamma_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$,
for every $u \in V_1$ & $(\mathrm{u}_2, v_2) \in E_2$

(iii) $(\mu_{\mathcal{B}_1L} \times \mu_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \mu_{\mathcal{A}_2L}(w)\}$

$(\mu_{\mathcal{B}_1U} \times \mu_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \mu_{\mathcal{A}_2U}(w)\}$

$(\eta_{\mathcal{B}_1L} \times \eta_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \eta_{\mathcal{A}_2L}(w)\}$

$(\eta_{\mathcal{B}_1U} \times \eta_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \eta_{\mathcal{A}_2U}(w)\}$

$(\gamma_{\mathcal{B}_1L} \times \gamma_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \gamma_{\mathcal{A}_2L}(\mathrm{w})\}$

$(\gamma_{\mathcal{B}_1U} \times \gamma_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \gamma_{\mathcal{A}_2U}(w)\}$,
for every $(\mathrm{u}_1, v_1) \in E_1$& $w \in V_2$

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Proposition 3.1 If G_1 and G_2 be strong IVPFGs, then the cartesian product $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is a strong interval valued picture fuzzy graph.

Proof: Let G_1 and G_2 be strong IVPFGs, then there exists $u_i, v_i \in E_i, i = 1, 2$ such that

$\mu_{B_iL}(u_i, v_i) = min(\mu_{A_iL}(u_i), \mu_{A_iL}(v_i))$; $\mu_{B_iU}(u_i, v_i) = min(\mu_{A_iU}(u_i), \mu_{A_iU}(v_i))$

$\eta_{B_iL}(u_i, v_i) = min(\eta_{A_iL}(u_i), \eta_{A_iL}(v_i))$; $\eta_{B_iU}(u_i, v_i) = min(\eta_{A_iU}(u_i), \eta_{A_iU}(v_i))$

$\gamma_{B_iL}(u_i, v_i) = max(\gamma_{A_iL}(u_i), \gamma_{A_iL}(v_i))$; $\gamma_{B_iU}(u_i, v_i) = max(\gamma_{A_iU}(u_i), \gamma_{A_iU}(v_i))$, for $i = 1, 2$

Let $E = \{(u, u_2)(u, v_2)/x \in V_1 \ \& \ (u_2, y_2) \in E_2\} \cup \{(u_1, w)(v_1, w)/w \in V_2 \& (u_1, v_1) \in E_1\}$

Consider $(u, u_2)(u, v_2) \in E$, we have

$(\mu_{B_1L} \times \mu_{B_2L})(u, u_2)(u, v_2) = min(\mu_{A_1L}(u), \mu_{B_2L}(u_2, v_2))$

$= \min(\mu_{A_1L}(u), \mu_{A_2L}(u_2), \mu_{A_2L}(v_2))$

Similarly, $(\mu_{B_1U} \times \mu_{B_2U})(u, u_2)(u, v_2) = min(\mu_{A_1U}(u), \mu_{B_2U}(u_2, v_2))$

$= \min(\mu_{A_1U}(u), \mu_{A_2U}(u_2), \mu_{A_2U}(v_2))$

$(\mu_{A_1L} \times \mu_{A_2L})(u_1, u_2) = min(\mu_{A_1L}(u_1), \mu_{A_2L}(u_2))$

$(\mu_{A_1U} \times \mu_{A_2U})(u_1, u_2) = min(\mu_{A_1U}(u_1), \mu_{A_2U}(u_2))$

$(\mu_{A_1L} \times \mu_{A_2L})(u_1, v_2) = min(\mu_{A_1L}(u_1), \mu_{A_2L}(v_2))$

$(\mu_{A_1U} \times \mu_{A_2U})(u_1, v_2) = min(\mu_{A_1U}(u_1), \mu_{A_2U}(v_2))$

$min((\mu_{A_1U} \times \mu_{A_2U})(u, u_2)(\mu_{A_1U} \times \mu_{A_2U})(u, v_2))$

$= min(min((\mu_{A_1U}(u), \mu_{A_2U}(u_2)), min((\mu_{A_1U}(u), \mu_{A_2U}(v_2)))$

$= min(\mu_{A_1U}(u), \mu_{A_2U}(u_2), \mu_{A_2U}(v_2))$

Hence, $(\mu_{B_1L} \times \mu_{B_2L})(u, u_2)(u, v_2) = min((\mu_{A_1L} \times \mu_{A_2L})(u, u_2)(\mu_{A_1L} \times \mu_{A_2L})(u, v_2))$

$(\mu_{B_1U} \times \mu_{B_2U})(u, u_2)(u, v_2) = min((\mu_{A_1U} \times \mu_{A_2U})(u, u_2)(\mu_{A_1U} \times \mu_{A_2U})(u, v_2))$

Similarly, we can show that

$(\eta_{B_1L} \times \eta_{B_2L})(u, u_2)(u, v_2) = min((\eta_{A_1L} \times \eta_{A_2L})(u, u_2)(\eta_{A_1L} \times \eta_{A_2L})(u, v_2))$

$(\eta_{B_1U} \times \eta_{B_2U})(u, u_2)(u, v_2) = min((\eta_{A_1U} \times \eta_{A_2U})(u, u_2)(\eta_{A_1U} \times \eta_{A_2U})(u, v_2))$

$(\gamma_{B_1L} \times \gamma_{B_2L})(u, u_2)(u, v_2) = min((\gamma_{A_1L} \times \gamma_{A_2L})(u, u_2)(\gamma_{A_1L} \times \gamma_{A_2L})(u, v_2))$

$(\gamma_{B_1U} \times \gamma_{B_2U})(u, u_2)(u, v_2) = min((\gamma_{A_1U} \times \gamma_{A_2U})(u, u_2)(\gamma_{A_1U} \times \gamma_{A_2U})(u, v_2))$

Hence, $G_1 \times G_2$ is strong IVPFG. This completes the proof.

Proposition 3.2 If $G_1 \times G_2$ is strong IVPFG, then at least G_1 *or* G_2 must be strong IVPFG.

Proof: Let G_1 and G_2 are not strong IVPFGs, then there exists $u_i, v_i \in E_i, i = 1, 2$ such that

$\mu_{B_iL}(u_i, v_i) < min(\mu_{A_iL}(u_i), \mu_{A_iL}(v_i))$; $\mu_{B_iU}(u_i, v_i) < min(\mu_{A_iU}(u_i), \mu_{A_iU}(v_i))$

$\eta_{B_iL}(u_i, v_i) < min(\eta_{A_iL}(u_i), \eta_{A_iL}(v_i))$; $\eta_{B_iU}(u_i, v_i) < min(\eta_{A_iU}(u_i), \eta_{A_iU}(v_i))$

$\gamma_{B_iL}(u_i, v_i) > max(\gamma_{A_iL}(u_i), \gamma_{A_iL}(v_i))$; $\gamma_{B_iU}(u_i, v_i) > max(\gamma_{A_iU}(u_i), \gamma_{A_iU}(v_i))$, for $i = 1, 2$

Let $E = \{(u, u_2)(u, v_2)/x \in V_1 \;\&\; (u_2, y_2) \in E_2\} \cup \{(u_1, w)(v_1, w)/w \in V_2 \& (u_1, v_1) \in E_1\}$

Consider $(u, u_2)(u, v_2) \in E$, we have

$$(\mu_{B_1L} \times \mu_{B_2L})(u, u_2)(u, v_2) = min(\mu_{A_1L}(u), \mu_{B_2L}(u_2, v_2))$$

$$< \min\,(\mu_{A_1L}(u), \mu_{A_2L}(u_2), \mu_{A_2L}(v_2))$$

Similarly, $(\mu_{B_1U} \times \mu_{B_2U})(u, u_2)(u, v_2) = min(\mu_{A_1U}(u), \mu_{B_2U}(u_2, v_2))$

$$< \min\,(\mu_{A_1U}(u), \mu_{A_2U}(u_2), \mu_{A_2U}(v_2))$$

$$(\mu_{A_1L} \times \mu_{A_2L})\,(u_1, u_2) = min(\mu_{A_1L}(u_1), \mu_{A_2L}(u_2))$$

$$(\mu_{A_1U} \times \mu_{A_2U})\,(u_1, u_2) = min(\mu_{A_1U}(u_1), \mu_{A_2U}(u_2))$$

$$(\mu_{A_1L} \times \mu_{A_2L})\,(u_1, v_2) = min(\mu_{A_1L}(u_1), \mu_{A_2L}(v_2))$$

$$(\mu_{A_1L} \times \mu_{A_2L})\,(u_1, v_2) = min(\mu_{A_1L}(u_1), \mu_{A_2L}(v_2))$$

$$min((\mu_{A_1U} \times \mu_{A_2U})(u, u_2)(\mu_{A_1U} \times \mu_{A_2U})(u, v_2))$$

$$= min(min((\mu_{A_1U}(u), \mu_{A_2U}(u_2)), min((\mu_{A_1U}(u), \mu_{A_2U}(v_2)))$$

$$= min(\mu_{A_1U}(u), \mu_{A_2U}(u_2), \mu_{A_2U}(v_2))$$

Hence, $(\mu_{B_1L} \times \mu_{B_2L})(u, u_2)(u, v_2) < min((\mu_{A_1L} \times \mu_{A_2L})(u, u_2)(\mu_{A_1L} \times \mu_{A_2L})(u, v_2))$

$$(\mu_{B_1U} \times \mu_{B_2U})(u, u_2)(u, v_2) < min((\mu_{A_1U} \times \mu_{A_2U})(u, u_2)(\mu_{A_1U} \times \mu_{A_2U})(u, v_2))$$

Similarly, we can show that

$$(\eta_{B_1L} \times \eta_{B_2L})(u, u_2)(u, v_2) < min((\eta_{A_1L} \times \eta_{A_2L})(u, u_2)(\eta_{A_1L} \times \eta_{A_2L})(u, v_2))$$

$$(\eta_{B_1U} \times \eta_{B_2U})(u, u_2)(u, v_2) < min((\eta_{A_1U} \times \eta_{A_2U})(u, u_2)(\eta_{A_1U} \times \eta_{A_2U})(u, v_2))$$

$$(\gamma_{B_1L} \times \gamma_{B_2L})(u, u_2)(u, v_2) < min((\gamma_{A_1L} \times \gamma_{A_2L})(u, u_2)(\gamma_{A_1L} \times \gamma_{A_2L})(u, v_2))$$

$$(\gamma_{B_1U} \times \gamma_{B_2U})(u, u_2)(u, v_2) < min((\gamma_{A_1U} \times \gamma_{A_2U})(u, u_2)(\gamma_{A_1U} \times \gamma_{A_2U})(u, v_2))$$

Hence, $G_1 \times G_2$ is not strong IVPFG, which is a contradiction. This completes the proof.

Remark 3.1 If G_1 is a Strong IVPFG and G_2 is not a strong IVPFG, then $G_1 \times G_2$ is need not strong IVPFG.

Example 3.3 Let $G_1 = (A_1, B_1)$ be a SIVPFG and G_2 is not a SIVPFG, then $G_1 \times G_2$ is a SIVPFG.

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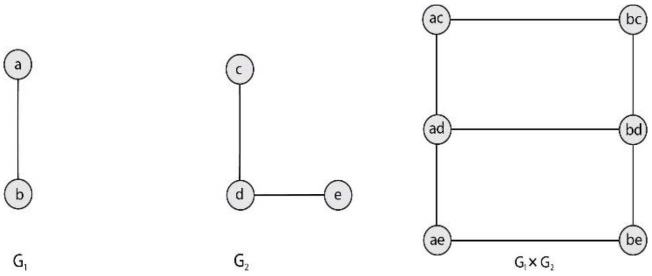

Fig 3: Strong IVPFG of $G_1 \times G_2$

Here $A_1 = \{$<a, [0.3, 0.4, 0.2], [0.2, 0.2, 0.5]>, <b, [0.6, 0.2, 0.1], [0.3, 0.5, 0.2]>$\}$ and $B_1 = \{$<ab, [0.3, 0.2, 0.2], [0.2, 0.2, 0.5]>$\}$. And let $G_2 = (A_2, B_2)$ is not a SIVPFG, where $A_2 = \{$<c, [0.4, 0.2, 0.4], [0.6, 0.2, 0.2]>, <d, [0.3, 0.4, 0.3], [0.4, 0.3, 0.2]>, <e, [0.5, 0.2, 0.3], [0.2, 0.1, 0.4]>$\}$ and $B_2 = \{$<cd, 0.2, 0.1, 0.5], [0.4, 0.2, 0.3]>, [de, [0.2, 0.2, 0.5], [0.1, 0.1, 0.5]$\}$, $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is a SIVPFG, where $A_1 \times A_2 = \{$<(a, c), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(b, c), [0.4, 0.2, 0.4], [0.3, 0.2. 0.2]>, <(a, d), [0.3, 0,4, 0.3], [0.2, 0.2, 0.5]>, <(b, d), [0.3, 0,2, 0.3], [0.3, 0.3, 0.2]>, <(a, e), [0.3, 0,2, 0.3], [0.2, 0.2, 0.5]>, <(b, e), [0.5, 0.2, 0.3], [0.2, 0.1, 0.4]>$\}$, $B_1 \times B_2 = \{$<(a, c)(b, c), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(a, d)(b, d), [0.3, 0.2, 0.3], [0.2, 0.2, 0.5]>, <(a, e)(b, e), [0.3, 0.2, 0.3], [0.2, 0.1, 0.5]>, <(a, c)(a, d), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(b, c)(b, d), [0.3, 0.2, 0.4], [0.3, 0.2, 0.2]>, <(a, d)(a, e), [0.3, 0.2, 0.3], [0.3, 0.1, 0.5]>, <(b, d)(b, e), [0.3, 0.2, 0.3], [0.2, 0.1, 0.4]>$\}$. In this example, G_1 be a SIVPFG and G_2 is not a SIVPFG, then $G_1 \times G_2$ is a SIVPFG.

Example 3.4 Let $G_1 = (A_1, B_1)$ be a SIVPFG where $A_1 = \{$<a, [0.2, 0.3, 0.4], [0.3, 0.2, 0.4]>, <b, [0.4, 0.3, 0.3], [0.2, 0.4, 0.3]>$\}$ and $B_1 = \{$<ab, [0.2, 0.3, 0.4], [0.2, 0.2, 0.4]>$\}$. Let $G_2 = (A_2, B_2)$ is not a SIVPFG, where $A_2 = \{$<c, [0.3, 0.3, 0.4], [0.4, 0.2, 0.3]>, <d, [0.2, 0.4, 0.3], [0.3, 0.2, 0.3]>, <e, [0.4, 0.2, 0.3], [0.3, 0.4, 0.3]>$\}$ and $B_2 = \{$<cd, 0.2, 0.2, 0.5], [0.3, 0.2, 0.4]>, [de, [0.2, 0.2, 0.5], [0.2, 0.2, 0.4]$\}$, $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is a SIVPFG, where $A_1 \times A_2 = \{$<(a, c), [0.1, 0.2, 0.5], [0.3, 0.1, 0.5]>, <(b, c), [0.2, 0.3, 0.5], [0.2, 0.1. 0.4]>, <(a, d), [0.2, 0.3, 0.4], [0.1, 0.2, 0.4]>, <(b, d), [0.1, 0.3, 0.4], [0.2, 0.2, 0.5]>, <(a, e), [0.2, 0.1, 0.6], [0.3, 0.2, 0.4]>, <(b, e), [0.3, 0.2, 0.3], [0.2, 0.3, 0.4]>$\}$, $B_1 \times B_2 = \{$<(a, c)(b, c), [0.1, 0.1, 0.6], [0.2, 0.1, 0.76]>, <(a, d)(b, d), [0.1, 0.2, 0.5], [0.1, 0.1, 0.5]>, <(a, e)(b, e), [0.2, 0.1, 0.6], [0.2, 0.2, 0.4]>, <(a, c)(a, d), [0.1, 0.2, 0.5], [0.1, 0.1, 0.5]>, <(b, c)(b, d), [0.1, 0.3, 0.5], [0.2, 0.1, 0.5]>, <(a, d)(a, e), [0.2, 0.1, 0.6], [0.1, 0.2, 0.4]>, <(b, d)(b, e), [0.1, 0.1,0.6], [0.2, 0.1, 0.6]>$\}$. In this example, G_1 be a SIVPFG and G_2 is not a SIVPFG, then $G_1 \times G_2$ is not a SIVPFG.

Definition: 3.4 The Composition of strong interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp graphs ${\mathcal{G}_1}^* = (V, E)$ and ${\mathcal{G}_2}^* = (V, E)$ is denoted
by $\mathcal{G}_1[\mathcal{G}_2] = (\mathcal{A}_1 \circ \mathcal{A}_2, \mathcal{B}_1 \circ \mathcal{B}_2)$ and defined as follows

(i) $(\mu_{\mathcal{A}_1 L} \circ \mu_{\mathcal{A}_2 L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{A}_1 L}(\mathrm{u}_1), \mu_{\mathcal{A}_2 L}(\mathrm{u}_2)\}$

$(\mu_{\mathcal{A}_1 U} \circ \mu_{\mathcal{A}_2 U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\mu_{\mathcal{A}_1 U}(\mathrm{u}_1), \mu_{\mathcal{A}_2 U}(\mathrm{u}_2)\}$

$(\eta_{\mathcal{A}_1 L} \circ \eta_{\mathcal{A}_2 L})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{A}_1 L}(\mathrm{u}_1), \eta_{\mathcal{A}_2 L}(\mathrm{u}_2)\}$

$(\eta_{\mathcal{A}_1 U} \circ \eta_{\mathcal{A}_2 U})(\mathrm{u}_1, \mathrm{u}_2) = \min\{\eta_{\mathcal{A}_1 U}(\mathrm{u}_1), \eta_{\mathcal{A}_2 U}(\mathrm{u}_2)\}$

$(\gamma_{\mathcal{A}_1 L} \circ \gamma_{\mathcal{A}_2 L})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{A}_1 L}(\mathrm{u}_1), \gamma_{\mathcal{A}_2 L}(\mathrm{u}_2)\}$

$(\gamma_{\mathcal{A}_1 U} \circ \gamma_{\mathcal{A}_2 U})(\mathrm{u}_1, \mathrm{u}_2) = \max\{\gamma_{\mathcal{A}_1 U}(\mathrm{u}_1), \gamma_{\mathcal{A}_2 U}(\mathrm{u}_2)\}$, for every $\mathrm{u}_1 \in V_1$ & $\mathrm{u}_2 \in \mathrm{V}_2$

(ii) $(\mu_{\mathcal{B}_1 L} \circ \mu_{\mathcal{B}_2 L})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{A}_1 L}(\mathrm{u}), \mu_{\mathcal{B}_2 L}(\mathrm{u}_2, v_2)\}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1834-1846
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$(\mu_{\mathcal{B}_1U} \circ \mu_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\mu_{\mathcal{A}_1U}(\mathrm{u}), \mu_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$

$(\eta_{\mathcal{B}_1L} \circ \eta_{\mathcal{B}_2L})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{A}_1L}(\mathrm{u}), \eta_{\mathcal{B}_2L}(\mathrm{u}_2, v_2)\}$

$(\eta_{\mathcal{B}_1U} \circ \eta_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \min\{\eta_{\mathcal{A}_1U}(\mathrm{u}), \eta_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$

$(\gamma_{\mathcal{B}_1L} \circ \gamma_{\mathcal{B}_2L})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{A}_1L}(\mathrm{u}), \gamma_{\mathcal{B}_2L}(\mathrm{u}_2, v_2)\}$

$(\gamma_{\mathcal{B}_1U} \circ \gamma_{\mathcal{B}_2U})(u, \mathrm{u}_2)(u, v_2) = \max\{\gamma_{\mathcal{A}_1U}(\mathrm{u}), \gamma_{\mathcal{B}_2U}(\mathrm{u}_2, v_2)\}$,
for every $u \in V_1$ & $(\mathrm{u}_2, v_2) \in E_2$

(iii) $(\mu_{\mathcal{B}_1L} \circ \mu_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \mu_{\mathcal{A}_2L}(w)\}$

$(\mu_{\mathcal{B}_1U} \circ \mu_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\mu_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \mu_{\mathcal{A}_2U}(w)\}$

$(\eta_{\mathcal{B}_1L} \circ \eta_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \eta_{\mathcal{A}_2L}(w)\}$

$(\eta_{\mathcal{B}_1U} \circ \eta_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \min\{\eta_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \eta_{\mathcal{A}_2U}(w)\}$

$(\gamma_{\mathcal{B}_1L} \circ \gamma_{\mathcal{B}_2L})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{B}_1L}(\mathrm{u}_1, v_1), \gamma_{\mathcal{A}_2L}(\mathrm{w})\}$

$(\gamma_{\mathcal{B}_1U} \circ \gamma_{\mathcal{B}_2U})(\mathrm{u}_1, w)(v_1, w) = \max\{\gamma_{\mathcal{B}_1U}(\mathrm{u}_1, v_1), \gamma_{\mathcal{A}_2U}(w)\}$
for every $w \in V_2$ & $(\mathrm{u}_1, v_1) \in E_1$

(iv) $(\mu_{\mathcal{B}_1L} \circ \mu_{\mathcal{B}_2L})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\mu_{\mathcal{A}_2L}(\mathrm{u}_2), \mu_{\mathcal{A}_2L}(v_2), \mu_{\mathcal{B}_1L}(\mathrm{u}_1, v_1)\}$

$(\mu_{\mathcal{B}_1U} \circ \mu_{\mathcal{B}_2U})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\mu_{\mathcal{A}_2U}(\mathrm{u}_2), \mu_{\mathcal{A}_2U}(v_2), \mu_{\mathcal{B}_1U}(\mathrm{u}_1, v_1)\}$

$(\eta_{\mathcal{B}_1L} \circ \eta_{\mathcal{B}_2L})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\eta_{\mathcal{A}_2L}(\mathrm{u}_2), \eta_{\mathcal{A}_2L}(v_2), \eta_{\mathcal{B}_1L}(\mathrm{u}_1, v_1)\}$

$(\eta_{\mathcal{B}_1U} \circ \eta_{\mathcal{B}_2U})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\eta_{\mathcal{A}_2U}(\mathrm{u}_2), \eta_{\mathcal{A}_2U}(v_2), \eta_{\mathcal{B}_1U}(\mathrm{u}_1, v_1)\}$

$(\gamma_{\mathcal{B}_1L} \circ \gamma_{\mathcal{B}_2L})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\gamma_{\mathcal{A}_2L}(\mathrm{u}_2), \gamma_{\mathcal{A}_2L}(v_2), \gamma_{\mathcal{B}_1L}(\mathrm{u}_1, v_1)\}$

$(\gamma_{\mathcal{B}_1U} \circ \gamma_{\mathcal{B}_2U})(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) = \min\{\gamma_{\mathcal{A}_2U}(\mathrm{u}_2), \gamma_{\mathcal{A}_2U}(v_2), \gamma_{\mathcal{B}_1U}(\mathrm{u}_1, v_1)\}$
For all $E^0 = E \cup \{(\mathrm{u}_1 \mathrm{u}_2)(v_1, \mathrm{v}_2) / (\mathrm{u}_1, v_1) \in E_1\ \mathrm{u}_2 \neq v_2\}$

The following theorem is stated without their proof.

Proposition 3.3 If G_1 and G_2 be the strong IVPFGs, then the Composition $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is a strong IVPFG.

Proposition 3.4 If $G_1[G_2]$ is strong IVPFG, then at least G_1 *or* G_2 must be strong IVPFG.

Example 3.5 Let $G_1 = (A_1, B_1)$ be a SIVPFG and $G_2 = (A_2, B_2)$ is not a SIVPFG, then $G_1[G_2]$ is not a SIVPFG.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1834-1846
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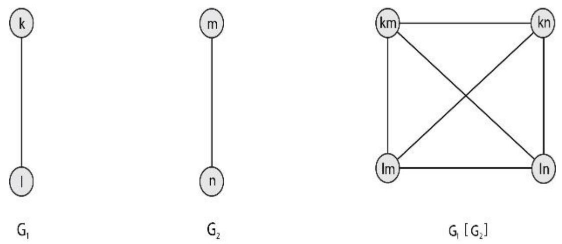

Fig 4: Strong IVPFG of $G_1[G_2]$

Here $A_1 = \{$<k, [0.6, 0.1, 0.3], [0.5, 0.2, 0.2]>, <l, [0.4, 0.2, 0.4], [0.3, 0.3, 0.2]>$\}$ and $B_1 = \{$<kl, [0.4, 0.1, 0.4], [0.3, 0.2, 0.2]>$\}$. Let $G_2 = (A_2, B_2)$ is not a SIVPFG, where $A_2 = \{$<m, [0.5, 0.3, 0.2], [0.4, 0.2, 0.3]>, <n, [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>$\}$ and $B_2 = \{$mn, [0.2, 0.1, 0.5], [0.2, 0.1, 0.6]>$\}$, $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is not a SIVPFG, where $A_1 \circ A_2 = \{$<(k, m), [0.5, 0.1, 0.3], [0.4, 0.2, 0.3]>, <(k, n), [0.3, 0.1, 0.4], [0.2, 0.2. 0.5]>, <(l, m), [0.4, 0,2, 0.4], [0.3, 0.2, 0.3]>, <(l, n), [0.3, 0,2, 0.4], [0.2, 0.2, 0.5]>$\}$, $B_1 \circ B_2 = \{$<(k, m)(k, n), [0.2, 0.1, 0.5], [0.1, 0.1, 0.4]>, <(l, m)(l, n), [0.2, 0.1, 0.6], [0.1, 0.2, 0.5]>, <(k, m)(l, m), [0.3, 0.1, 0.5], [0.2, 0.2, 0.6]>, <(k, n)(l, n), [0.2, 0.1, 0.5], [0.1, 0.1, 0.6]>, <(k, m)(l, n), [0.3, 0.1, 0.5], [0.2, 0.2, 0.5]>, <(k, n)(l, m), [0.3, 0.1, 0.5], [0.2, 0.2, 0.6]>$\}$. In this example, G_1 be a SIVPFG and G_2 is not a SIVPFG, then $G_1[G_2]$ is not a SIVPFG.

Example 3.6 Let $G_1 = (A_1, B_1)$ be a SIVPFG where $A_1 = \{$<k, [0.4, 0.2, 0.3], [0.3, 0.2, 0.4]>, <l, [0.3, 0.3, 0.4], [0.2, 0.3, 0.3]>$\}$ and $B_1 = \{$<kl, [0.3, 0.2, 0.4], [0.2, 0.2, 0.4]>$\}$. Let $G_2 = (A_2, B_2)$ is not a SIVPFG, where $A_2 = \{$<m, [0.4, 0.3, 0.2], [0.4, 0.2, 0.3]>, <n, [0.3, 0.2, 0.5], [0.3, 0.2, 0.5]>$\}$ and $B_2 = \{$mn, [0.2, 0.1, 0.5], [0.2, 0.1, 0.6]>$\}$, $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is not a SIVPFG, where $A_1 \circ A_2 = \{$<(k, m), [0.4, 0.2, 0.3], [0.4, 0.2, 0.4]>, <(k, n), [0.3, 0.2, 0.4], [0.2, 0.2. 0.5]>, <(l, m), [0.4, 0,2, 0.4], [0.3, 0.2, 0.3]>, <(l, n), [0.3, 0,2, 0.4], [0.2, 0.2, 0.5]>$\}$, $B_1 \circ B_2 = \{$<(k, m)(k, n), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(l, m)(l, n), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(k, m)(l, m), [0.4, 0.1, 0.4], [0.3, 0.2, 0.3]>, <(k, n)(l, n), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(k, m)(l, n), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>, <(k, n)(l, m), [0.3, 0.2, 0.4], [0.2, 0.2, 0.5]>$\}$. In this example, G_1 be a SIVPFG and G_2 is not a SIVPFG, then $G_1[G_2]$ is a SIVPFG.

Definition 3.5 The union of $G_1 \cup G_2$ of two strong IVPFG G_1 and G_2 of the graph ${G_1}^*$ and ${G_2}^*$ is defined as follows

(i) $(\mu_{A_1L} \cup \mu_{A_2L})(x) = \mu_{A_1L}(x), if\ x \in V_1$ and $if\ x \notin V_2$

$(\mu_{A_1L} \cup \mu_{A_2L})(x) = \mu_{A_2L}(x),\ if\ x \in V_2$ and $if\ x \notin V_1$

$(\mu_{A_1L} \cup \mu_{A_2L})(x) = \max(\mu_{A_1L}(x), \mu_{A_2L}(x)), if\ x \in V_1 \cup V_2$

$(\eta_{A_1L} \cup \eta_{A_2L})(x) = (\eta_{A_1L})(x), if\ x \in V_1$ and $if\ x \notin V_2$

$(\eta_{A_1L} \cup \eta_{A_2L})(x) = (\eta_{A_2L})(x), if\ x \in V_2$ and $if\ x \notin V_1$

$(\eta_{A_1L} \cup \eta_{A_2L})(x) = \max(\eta_{A_1L}, \eta_{A_2L})(x), if\ x \in V_1 \cup V_2$

$(\gamma_{A_1L} \cup \gamma_{A_2L})(x) = (\gamma_{A_1L}), if\ x \in V_1$ and $if\ x \notin V_2$

$(\gamma_{A_1L} \cup \gamma_{A_2L})(x) = (\gamma_{A_2L}), if\ x \in V_2$ and $if\ x \notin V_1$

$(\gamma_{A_1L} \cup \gamma_{A_2L})(x) = \min(\gamma_{A_1L}, \gamma_{A_2L})(x), if\ x \in V_1 \cup V_2$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1834-1846
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$(\mu_{A_1U} \cup \mu_{A_2U})(x) = \mu_{A_1U}(x), if\ x \in V_1$ and $if\ x \notin V_2$

$(\mu_{A_1U} \cup \mu_{A_2U})(x) = \mu_{A_2U}(x),\ if\ x \in V_2$ and $if\ x \notin V_1$

$(\mu_{A_1U} \cup \mu_{A_2U})(x) = \max(\mu_{A_1U}(x), \mu_{A_2U}(x)), if\ x \in V_1 \cap V_2$

$(\eta_{A_1U} \cup \eta_{A_2U})(x) = (\eta_{A_1U})(x), if\ x \in V_1$ and $if\ x \notin V_2$

$(\eta_{A_1U} \cup \eta_{A_2U})(x) = (\eta_{A_2U})(x), if\ x \in V_2$ and $if\ x \notin V_1$

$(\eta_{A_1U} \cup \eta_{A_2U})(x) = \max(\eta_{A_1U}, \eta_{A_2U})(x), if\ x \in V_1 \cap V_2$

$(\gamma_{A_1U} \cup \gamma_{A_2U})(x) = (\gamma_{A_1U}), if\ x \in V_1 \& if\ x \notin V_2$

$(\gamma_{A_1U} \cup \gamma_{A_2U})(x) = (\gamma_{A_2U}), if\ x \in V_2$ and $if\ x \notin V_1$

$(\gamma_{A_1U} \cup \gamma_{A_2U})(x) = \min(\gamma_{A_1U}, \gamma_{A_2U})(x), if\ x \in V_1 \cap V_2$

(ii) $(\mu_{B_1L} \cup \mu_{B_2L})(x, y) = \mu_{B_1L}(x, y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$(\mu_{B_1L} \cup \mu_{B_2L})(x, y) = \mu_{B_2L}(x, y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$(\mu_{B_1L} \cup \mu_{B_2L})(xy) = \max(\mu_{B_1L} \cup \mu_{B_2L})(xy),\ if\ x, y \in E_1 \cap E_2$

$(\eta_{B_1L} \cup \eta_{B_2L})(xy) = \eta_{B_1L}(x, y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$(\eta_{B_1L} \cup \eta_{B_2L})(xy) = \eta_{B_2L}(x, y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$(\eta_{B_1L} \cup \eta_{B_2L})(xy) = \max(\eta_{B_1L} \cup \eta_{B_2L})(xy),\ if\ xy \in E_1 \cap E_2$

$(\gamma_{B_1L} \cup \gamma_{B_2L})(xy) = \gamma_{B_1L}(x, y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$(\gamma_{B_1L} \cup \gamma_{B_2L})(xy) = \gamma_{B_2L}(x, y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$(\gamma_{B_1L} \cup \gamma_{B_2L})(xy) = \min(\gamma_{B_1L} \cup \gamma_{B_2L})(xy),\ if\ xy \in E_1 \cap E_2$

$(\mu_{B_1U} \cup \mu_{B_2U})(x, y) = \mu_{B_1U}(x, y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$(\mu_{B_1U} \cup \mu_{B_2U})(x, y) = \mu_{B_2U}(x, y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$(\mu_{B_1U} \cup \mu_{B_2U})(xy) = \max(\mu_{B_1U} \cup \mu_{B_2U})(xy),\ if\ x, y \in E_1 \cap E_2$

$(\eta_{B_1U} \cup \eta_{B_2U})(xy) = \eta_{B_1U}(x, y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$(\eta_{B_1U} \cup \eta_{B_2U})(xy) = \eta_{B_2U}(x, y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$\left(\eta_{B_1U} \cup \eta_{B_2U}\right)(\mathrm{xy}) = \max\left(\eta_{B_1U} \cup \eta_{B_2U}\right)(xy),\ if\ xy \in E_1 \cap E_2$

$\left(\gamma_{B_1U} \cup \gamma_{B_2U}\right)(\mathrm{xy}) = \gamma_{B_1U}(x,y)$, if $x, y \in E_1$ and $x, y \notin E_2$

$\left(\gamma_{B_1U} \cup \gamma_{B_2U}\right)(\mathrm{xy}) = \gamma_{B_2U}(x,y)$, if $x, y \in E_2$ and $x, y \notin E_1$

$\left(\gamma_{B_1U} \cup \gamma_{B_2U}\right)(\mathrm{xy}) = \min\left(\gamma_{B_1U} \cup \gamma_{B_2U}\right)(xy),\ if\ xy \in E_1 \cap E_2$

Proposition 3.5 If G_1 and G_2 be the strong IVPFGs, then $G_1 \cup G_2$ is need not be strong IVPFG.

Example 3.7 Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be a SIVPFG, then $G_1 \cup G_2$ is not a SIVPFG

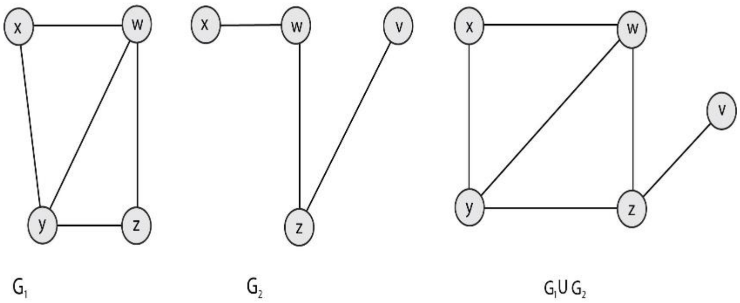

Fig 5: $G_1 \cup G_2$ is not a SIVPFG

Here $A_1 =$ {<w, [0.3, 0.2, 0.4], [0.2, 0.1, 0.5]>, <x, [0.1, 0.4, 0.4], [0.2, 0.3, 0.4]>, <y, [0.2, 0.3, 0.3], [0.4, 0.2, 0.3]>, <z, [0.2, 0.4, 0.2], [0.3, 0.1, 0.4]>} and $B_1 =$ {<wx, [0.1, 0.2, 0.4], [0.2, 0.1, 0.5]>, <xy, [0.1, 0.3, 0.4], [0.2, 0.2, 0.4]>, <yz, [0.2, 0.3, 0.2], [0.3, 0.1, 0.4]>, <wy, [0.2, 0.2, 0.4], [0.2, 0.1, 0.5]>, <wz, [0.2, 0.2, 0.4], [0.2, 0.1, 0.5]>}. Let $G_2 = (A_2, B_2)$ is a SIVPFG, where $A_2 =$ {<v, [0.1, 0.3, 0.5], [0.2, 0.3, 0.4]>, <w, [0.3, 0.3, 0.4], [0.4, 0.3, 0.2]>, <x, [0.2, 0.3, 0.5], [0.2, 0.1, 0.4]>, <z, [0.2, 0.2, 0.3], [0.1, 0.1, 0.5]>} and $B_2 =$ {<xw, [0.2, 0.3, 0.4], [0.2, 0.1, 0.4]>, <vz, [0.1, 0.2, 0.5], [0.1, 0.1, 0.5]>, <wz, [0.2, 0.2, 0.4], [0.1, 0.1, 0.5]>}, $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ is not a SIVPFG, where $A_1 \cup A_2 =$ {<x, [0.1, 0.3, 0.5], [02, 0.3, 0.4]>, <w, [0.3, 0.2, 0.4], [0.2, 0.1. 0.5]>, <z, [0.2, 0,2, 0.3], [0.1, 0.1, 0.5]>, <y, [0.2, 0.3, 0.3], [0.4, 0.2, 0.3]>, <v, [0.1, 0.3, 0.5], [0.2, 0.3, 0.4]>}, $B_1 \cup B_2 =$ {<xy, [0.1, 0.3, 0.4], [0.2, 0.2, 0.4]>, <yz, [0.2, 0.3, 0.2], [0.3, 0.1, 0.4]>, <vz, [0.1, 0.2, 0.5], [0.1, 0.1, 0.5]>, <wy, [0.2, 0.2, 0.4], [0.2, 0.1, 0.5]>, <xw, [0.1, 0.2, 0.4], [0.2, 0.1, 0.5]>, <wz, [0.2, 0.2, 0.4], [0.1, 0.1, 0.5]>}. In this example, G_1 and G_2 be a SIVPFG, then $G_1 \cup G_2$ is not a SIVPFG.

Definition 3.6 The join of two strong IVPFG G_1 and G_2 of the graph ${G_1}^*$ and ${G_2}^*$ is denoted by $G_1 + G_2$ and is defined as follows

(i) $\left(\mu_{\mathcal{A}_1L} + \mu_{\mathcal{A}_2L}\right)(\mathrm{x}) = \left(\mu_{\mathcal{A}_1L} \cup \mu_{\mathcal{A}_2L}\right)(\mathrm{x})$

$\left(\eta_{\mathcal{A}_1L} + \eta_{\mathcal{A}_2L}\right)(\mathrm{x}) = \left(\eta_{\mathcal{A}_1L} \cup \eta_{\mathcal{A}_2L}\right)(\mathrm{x})$

$\left(\gamma_{\mathcal{A}_1L} + \gamma_{\mathcal{A}_2L}\right)(\mathrm{x}) = \left(\gamma_{\mathcal{A}_1L} \cup \gamma_{\mathcal{A}_2L}\right)(\mathrm{x}),\ if\ x \in V_1 \cup V_2$

$\left(\mu_{\mathcal{A}_1U} + \mu_{\mathcal{A}_2U}\right)(\mathrm{x}) = \left(\mu_{\mathcal{A}_1U} \cup \mu_{\mathcal{A}_2U}\right)(\mathrm{x})$

$\left(\eta_{\mathcal{A}_1U} + \eta_{\mathcal{A}_2U}\right)(\mathrm{x}) = \left(\eta_{\mathcal{A}_1U} \cup \eta_{\mathcal{A}_2U}\right)(\mathrm{x})$

$\left(\gamma_{\mathcal{A}_1U} + \gamma_{\mathcal{A}_2U}\right)(\mathrm{x}) = \left(\gamma_{\mathcal{A}_1U} \cup \gamma_{\mathcal{A}_2U}\right)(\mathrm{x}),\ if\ x \in V_1 \cup V_2$

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(ii) $\left(\mu_{\mathcal{B}_1L}+\mu_{\mathcal{B}_2L}\right)(\mathrm{xy}) = (\mu_{\mathcal{B}_1L}\cup\mu_{\mathcal{B}_2L})(xy)$

$\left(\mu_{\mathcal{B}_1U}+\mu_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \left(\mu_{\mathcal{B}_1U}\cup\mu_{\mathcal{B}_2U}\right)(xy),\ if\ xy\in E_1\cap E_2$

$\left(\eta_{\mathcal{B}_1L}+\eta_{\mathcal{B}_2L}\right)(\mathrm{xy}) = (\eta_{\mathcal{B}_1L}\cup\eta_{\mathcal{B}_2L})(xy)$

$\left(\eta_{\mathcal{B}_1U}+\eta_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \left(\eta_{\mathcal{B}_1U}\cup\eta_{\mathcal{B}_2U}\right)(xy),\ if\ xy\in E_1\cap E_2$

$\left(\gamma_{\mathcal{B}_1L}+\gamma_{\mathcal{B}_2L}\right)(\mathrm{xy}) = (\gamma_{\mathcal{B}_1L}\cup\gamma_{\mathcal{B}_2L})(xy)$

$\left(\gamma_{\mathcal{B}_1U}+\gamma_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \left(\gamma_{\mathcal{B}_1U}\cup\gamma_{\mathcal{B}_2U}\right)(xy),\ if\ xy\in E_1\cap E_2$

(iii) $\left(\mu_{\mathcal{B}_1L}+\mu_{\mathcal{B}_2L}\right)(\mathrm{xy}) = \min(\mu_{\mathrm{A}_1L}(x),\ \mu_{\mathrm{A}_2L}(y))$

$\left(\mu_{\mathcal{B}_1U}+\mu_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \min(\mu_{\mathrm{A}_1U}(x),\ \mu_{\mathrm{A}_2U}(y))$

$\left(\eta_{\mathcal{B}_1L}+\eta_{\mathcal{B}_2L}\right)(\mathrm{xy}) = \min(\eta_{\mathrm{A}_1L}(x),\eta_{A_2L}(y))$

$\left(\eta_{\mathcal{B}_1U}+\eta_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \min(\eta_{\mathrm{A}_1U}(x),\eta_{A_2U}(y))$

$\left(\gamma_{\mathcal{B}_1L}+\gamma_{\mathcal{B}_2L}\right)(\mathrm{xy}) = \max(\gamma_{\mathrm{A}_1L}(x),\ \gamma_{\mathrm{A}_2L}(y)$

$\left(\gamma_{\mathcal{B}_1U}+\gamma_{\mathcal{B}_2U}\right)(\mathrm{xy}) = \max(\gamma_{\mathrm{A}_1U}(x),\gamma_{\mathrm{A}_2U}(y))$

for every $xy\in E'$, where E' is the set of all edges joining the vertices of V_1 and V_2.

Proposition 3.6 If G_1 and G_2 be the strong IVPFGs, then the join G_1+G_2 is also a strong IVPFG.

Proof: Let G_1 and G_2 be strong IVPFGs of ${G_1}^*$ and ${G_2}^*$ respectively.
Let (u, v) ∈ E',

$\left(\mu_{\mathrm{B}_1L}+\mu_{\mathrm{B}_2L}\right)(\mathrm{uv}) = \min\,(\mu_{\mathrm{A}_1L}(u),\mu_{\mathrm{A}_2L}(v))$

$= \min\,((\mu_{\mathrm{A}_1L}\cup\mu_{A_2L})(u),(\mu_{\mathrm{A}_1L}\cup\mu_{A_2L})(v))$

$= \min\,((\mu_{\mathrm{A}_1L}+\mu_{A_2L})(u),(\mu_{\mathrm{A}_1L}+\mu_{A_2L})(v))$

$\left(\mu_{\mathrm{B}_1U}+\mu_{\mathrm{B}_2U}\right)(\mathrm{uv}) = \min\,(\mu_{\mathrm{A}_1U}(u),\mu_{\mathrm{A}_2U}(v))$

$= \min\,((\mu_{\mathrm{A}_1U}\cup\mu_{A_2U})(u),(\mu_{\mathrm{A}_1U}\cup\mu_{A_2U})(v))$

$= \min\,((\mu_{\mathrm{A}_1U}+\mu_{A_2U})(u),(\mu_{\mathrm{A}_1U}+\mu_{A_2U})(v))$

$\left(\eta_{\mathrm{B}_1L}+\eta_{\mathrm{B}_2L}\right)(\mathrm{uv}) = \min\,(\eta_{\mathrm{A}_1L}(u),\eta_{\mathrm{A}_2L}(v))$

$= \min\,((\eta_{\mathrm{A}_1L}\cup\eta_{A_2L})(u),(\eta_{\mathrm{A}_1L}\cup\eta_{A_2L})(v))$

$$= \min\left((\eta_{A_1L} + \eta_{A_2L})(u), (\eta_{A_1L} + \eta_{A_2L})(v)\right)$$

$$\left(\eta_{B_1U} + \eta_{B_2U}\right)(\mathrm{uv}) = \min\left(\eta_{A_1U}(u), \eta_{A_2U}(v)\right)$$

$$= \min\left((\eta_{A_1U} \cup \eta_{A_2U})(u), (\eta_{A_1U} \cup \eta_{A_2U})(v)\right)$$

$$= \min\left((\eta_{A_1U} + \eta_{A_2U})(u), (\eta_{A_1U} + \eta_{A_2U})(v)\right)$$

$$\left(\gamma_{B_1L} + \gamma_{B_2L}\right)(\mathrm{uv}) = \max\left(\gamma_{A_1L}(u), \gamma_{A_2L}(v)\right)$$

$$= \max\left((\gamma_{A_1L} \cup \gamma_{A_2L})(u), (\gamma_{A_1L} \cup \gamma_{A_2L})(v)\right)$$

$$= \max\left((\gamma_{A_1L} + \gamma_{A_2L})(u), (\gamma_{A_1L} + \gamma_{A_2L})(v)\right)$$

$$\left(\gamma_{B_1U} + \gamma_{B_2U}\right)(\mathrm{uv}) = \max\left(\gamma_{A_1U}(u), \gamma_{A_2U}(v)\right)$$

$$= \max\left((\gamma_{A_1U} \cup \gamma_{A_2U})(u), (\gamma_{A_1U} \cup \gamma_{A_2U})(v)\right)$$

$$= \max\left((\gamma_{A_1U} + \gamma_{A_2U})(u), (\gamma_{A_1U} + \gamma_{A_2U})(v)\right)$$

IV Conclusion

This paper has discussed a subclass interval valued picture fuzzy graph called a strong interval valued picture fuzzy graph. Also, introduced some operations, such as Cartesian product, Composition, union and join of two strong interval valued picture fuzzy graphs, along with the proofs and illustrated with some examples.

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