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Maximal Dominating Cardinality in Hamiltonian Graph

Subha vidhya[1] , P.Tharaniya[2] and G.Jayalalitha[3]

1,2 Research Scholar, Department of Mathematics,
VELS Institute of Science, Technology and Advanced Studies, Chennai, India
m.subhavidya@gmail.comtharamanisharmi@gmail.com.

3 Professor, Department of Mathematics,
VELS Institute of Science, Technology and Advanced studies, Chennai, India
g.jayalalithamaths.sbs@velsuniv.ac.in

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ABSTRACT

The goal of this article provides the common formula for Minimal Domination Cardinality for all Hamiltonian Path or Hamiltonian Circuit. It is explained the way of structures and characteristics of General Hamiltonian Path or Hamiltonian Circuit. It invents the result that all the Hamiltonian Circuit from the given graph should be Cycle Graph. These derived formulae are common for all graphs those have Hamiltonian path or Hamiltonian Circuit. This formula is used to find Maximal Dominating Cardinality in a simple manner. Even though complicated graph also which has Hamiltonian Circuit is evaluating the Cardinality of Maximal Dominating Set is very easy way.

Keywords: Minimal Dominating Set, Induction Method, Graph Partition.

AMS Classification key: 05C05, 05C70, 05C69

1. INTRODUCTION - GRAPH

Theory of Graph is the new modern study about graphs in Mathematics [2]. Graphs has been characterized under their mathematical structures that it has been used the relation between vertices and edges. Vertex or node is the fundamental unit of graphs which is represented by dots or circles and edges are represented by straight or bended lines from one vertex to other vertex. Graphs are categorized into many types of graphs such as Null Graph, Trivial Graph, Complete Graph, Cycle Graph, Euler Graph, and Hamiltonian Graph and so on. which is depending on their graphical structures, adjacency and incidence conditions with vertices as well as edges and degree of the vertices[1].

2. PRELIMINARIES

This Paper analyses the concept of Hamiltonian Path, Hamiltonian Circuit, Maximal Matching Set, Maximal Matching Cardinality and Cycle Graph

2.1 HAMILTONIAN PATH

A graph is said to be Hamiltonian Path if it has the following characteristics[4]

(i) the path is passing through all the vertices exactly once

(ii) the path should consider all the vertices of the given graph

(iii) starting and ending vertices of the path need not be same

2.2 HAMILTONIAN CIRCUIT

A graph is said to be Hamiltonian circuit if it has formed the circuit that following the characteristics

(i) passing through all the vertices exactly once

(ii) without missing any vertices of the given graph through the path

(iii) The circuit should reach the starting vertex who is first vertex of the path [7]

2.3 MAXIMUM DOMINATING SET

The selection of non-touching vertices from the graph is called as largest domination set. Total edges of this domination set are called as Domination Number[3]. The collection of maximum number of non-ad joint vertices in the dominating set is called maximum dominating set. It is mentioned by $\boldsymbol{D(G)}$**.**

2.4 MAXIMAL DOMINATING SET

The collection of low level number of non-ad joint vertices for the Dominating Set is called Maximal Dominating Set. These vertices are connected with the remaining vertices in the graph. It is mentioned by *D '(G).* It is also called by Maximal Independent Set.

2.5 DOMINATION CARDINALITY

The minimum number of nodes included in the domination set is called as domination number. It is named by $\propto (\boldsymbol{G})$**.**

2.6 CONNECTED DOMINATING SET

A Connected Subgraph on dominating set is called as Connected Dominating Set[5]. Connected Dominating Set C*D* is the collection of vertices of a graph *G*, such that every vertex in CD- *u* is adjacent to at least one vertex in C*D* and the sub-graph <*CD*> induced by the set *CD* is connected. Connected Dominating Set is a dominating set in G which has induced the vertices on connected subgraph.[6]

2.6 NODE CONNECTIVITY

Let G be a joined graph. The vertex connectivity of G is the collection of vertices in which removal of these vertices makes the graph to be non-connected graph[8]. The node connectivity of a connected graph is denoted by L (G).

If G is a disconnected graph, then L (G) =0.

Note

- The complete graph k_n is to be disconnected if removing n-1 vertices from the graph. It gives results as Trivial Graph. Hence $\lambda (k_n) = n - 1$.
- If vertex is cut vertex then vertex connectivity of a graph is one
- In a Path, Vertex connectivity is one
- For cycle graph C_n, Vertex connectivity is two.

2.7 **CYCLE GRAPH**

This graph is connected closed path graph. It has same element of vertices and edges. All the nodes are joined one by one like path wise through the edges. Path starting and closing vertex are same. There is no edges are crossed together. It has common degree itself. Each vertex has two degree in common. Alternative iteration, edges are increased by one. At the initial stage of cycle graph is started with three edges. Total number of edges and vertices are equal for entire cycle graph. It is denoted by C_n. 'n' is the count of vertices of cycle graph. [12]

2.7.1 Maximal Dominating Cardinality for Cycle Graph[9]

It is closed and connected path. It is denoted by C_n where $n \geq 3$. Cycle Graphs have an equal count of vertices and edges. The general formulae (1) for Maximal Dominating Cardinality are given for all cycle graphs. Maximal Dominating Cardinality is evaluated by Formula (1) by using the number of vertices in the given graph.

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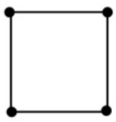
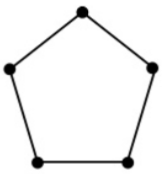
Maximal Dominating Cardinality= $\begin{Bmatrix} \left\lfloor \frac{n}{3} \right\rfloor & \textbf{\textit{if n is multiple of 3}} \\ \left\lfloor \frac{n}{3} \right\rfloor + 1 & \textbf{\textit{Otherwise}} \end{Bmatrix}$ ***(1)***

Here modulus attains the whole number only.

2.8 CALCULATION OF MAXIMAL DOMINATING CARDINALITY FOR CYCLE GRAPH

S. No.	Cycle Graph	No. of Vertices n	Maximal Domination Cardinality
1		3	1
2		4	2
3		5	2

Note: All Hamiltonian circuit are cycle graph

3. MAXIMAL DOMINATING CARDINALITY OF ALL HAMILTONIAN PATH OR HAMILTONIAN CIRCUIT

THEOREM 3.1

If the graph G(n,e) is said to be Hamiltonian Path or Hamiltonian Circuit then it has $\left\lfloor \frac{n}{3} \right\rfloor + 1$ number of vertices in Maximal dominating set.

Proof

This theorem can be proved by mathematical induction method. Let consider any graph with n vertices and e edges. Select a path or cycle who is incident with all the given vertices with 2 degree through the non-touching edges from the given path. If the path is ended with the initial vertex of the path, then it is considered as a cycle.[13]

Step 1 Let to prove the theorem by n=2. If n=2, it should be a path with 1 degree.

Maximal Dominating Cardinality = $\left\lfloor \frac{n}{3} \right\rfloor + 1 = \left\lfloor \frac{1}{3} \right\rfloor + 1 = 1$

If n=3, it should be path or cycle

Case (i)

If it is cycle, it is 2 regular graph. Maximal Dominating Cardinality = $\left\lfloor \frac{3}{3} \right\rfloor = 1$

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Case (ii)

If it is a path, then Maximal Dominating Cardinality = $\left\lfloor\frac{3}{3}\right\rfloor = 1$

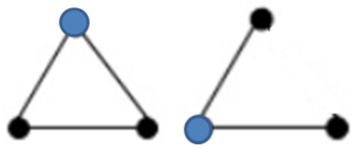

Figure 1: Maximal Dominating Set of 3 vertex Graph

Step 2

Let us assume that the theorem is proved for n=k. If the graph has k vertices then Maximal Dominating Cardinality =

$$\begin{Bmatrix} \left\lfloor\frac{k}{3}\right\rfloor & \text{n is multiple of 3} \\ \left\lfloor\frac{k}{3}\right\rfloor + 1 & \text{Otherwise} \end{Bmatrix}$$

Step 3

To Prove that the statement is true for n=k+1. The graph with k+1 vertices is partialed into two component[14]. The first component has k vertices and second component has one vertex. By the assumption, the first component has

$$\text{Maximal Domination Cardinality} = \begin{Bmatrix} \left\lfloor\frac{k}{3}\right\rfloor & \text{if n is multiple of 3} \\ \left\lfloor\frac{k}{3}\right\rfloor + 1 & \text{Otherwise} \end{Bmatrix}$$

The second component has single vertex, definitely it should be considered for dominating set. Therefore Maximal Domination Cardinality is one for the second component. Both components should be connected and want to create a path or cycle[11]. Let us proved it by an example from the graph Figure (2), it has k+1=5 vertices and

Maximal Domination Cardinality=$\left\lfloor\frac{5}{3}\right\rfloor + 1 = 2$

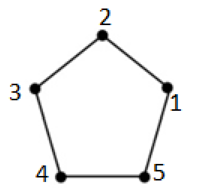

Figure 2: 5 vertex Cycle Graph

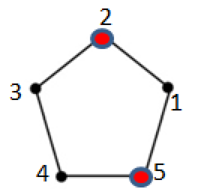

Figure 3: Maximal Dominated Vertex

The figure has been partialled into two components Figure (4) . The first component has two maximal matching vertices and the second component has only one vertex therefore it is considered as the maximal matching vertex. Totally the two components have three vertices in Maximal Dominating Set. It is shown as Figure (5). But it is contrary to our assumption. The first component has four vertices like 1,5,4 and 3. Vertex 1 and 3 has single degree. Other two vertices 4 and 5 have two degree. Possibility of vertex selection is 1&3, 1&4 and 3&5. But 4& 5 are not selected for Maximal Dominating Cardinality vertex because of the adjacency condition[10].

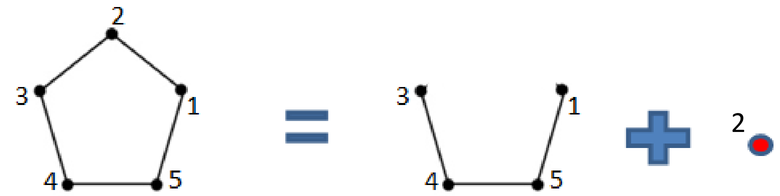

Figure 4: Partialled Component of k+1=5 vertex Graph

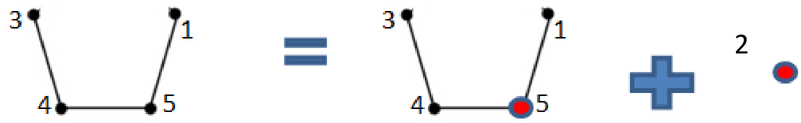

Figure 5 : Maximal Dominating Cardinality of k+1=5 vertex Graph

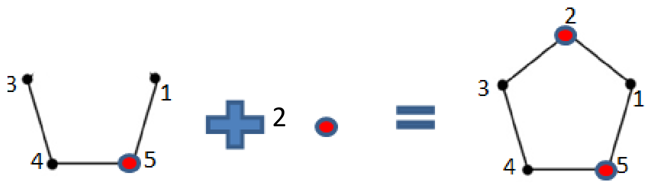

Figure 6: Maximal Dominating Cardinality for k+1 vertex through Hamiltonian Circuit

Vertex 2 has been joined to the first component which is making a path or cycle. Vertex 2 has been joined with the vertices 1 and 3. It is creating a cycle[14]. By the adjacency condition, there is only one vertex selected for Maximal Dominating set among the three vertices 1,2 and 3. Finally only two vertices are selected for Maximal Matching Set. Suppose it is joining the vertex 1 or 3, it may create a path[15]. It is shown in Figure (6)

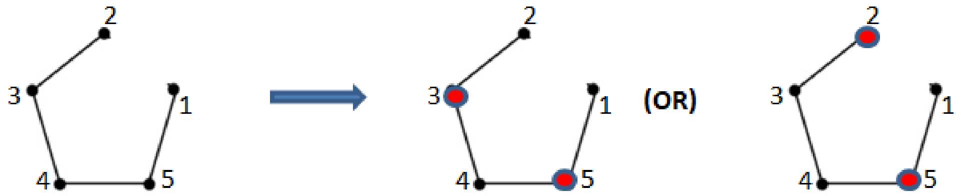

Figure 6: Maximal Dominating Cardinality for k+1 vertex through Hamiltonian Path

Therefore The threoem is true for n= k+1. Induction method is proved. Therefore the theorem is proved for all n.

4. CONCLUSION

In this paper apply the concept of Maximal Dominating in Hamiltonian Path or Hamiltonian Circuit. Adjacency of vertices as well as edges has been analysed. It is derived two General Formula for the calculation of Maximum Matching Cardinality. Hamiltonian Path or Hamiltonian Circuit is found from the given graph by the satisfaction of all the conditions. Hamiltonian circuit should be a cyle graph. This statement is proved by an example. By using the general formula of Maximal Dominating Cardinality, it is very easy to solve complicated large graph.

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