JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1853-1858
https://publishoa.com
ISSN: 1309-3452

Statistical Fractal Analysis in Perforamnce of Student's Mark

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Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

ABSTRACT

The Arithmetical Experimentation in Students performance which depends on decision theory. Statistics permit instructor to make out student's performance by utilizing data visualization and collate teaching procedure by employing hypothesis test. The Enactment of Lacunarity implicit that Random sample is a Statistical Fractal. The Random sample X and Y are totally disconnected but selfsame in shape to manifest self – similarity.

KEYWORDS: Sampling distribution, t – test statistics, Lacunarity, Statistical Fractal.

1. INTRODUCTION

The word statistics have been acquire from Latin word "Status" or the Italian word

"Statista" content of these words is "Political State" or a Government. Previously, the statistics was cast off by rulers. The Utilization of statistics was very finite but rulers and kings require data about coastline, agriculture, commerce, population of their states to evaluate their military potential, their wealth, taxation and other exposure of government. Statistics directly means numerical data, and is field of math that generally give out with data mining, table of content, and simplification of mathematical data. It is literally a form of statistical scanning those assets disparate measurable models to generate a set of initial data or research of real life. It is a domain of operative mathematics deal with statistic stack analysis, simplification, and display. The population tally accords with in what way result can be castoff to answer complicated hurdles. A Certain mankind think over that statistics to be a clear cut mathematical science rather than an outgrowth of math. Statistics is also stated to as appearing at termination from the selection data that is collected using prospects or trials [9]. Different categories such as psychology, sociology, geology, probability, and so on also use enumeration to activity. Computational data is a vital subspace of the regulation of statistics. Statistical thinkers research and enrich statistical technique with mathematics, and statistical research often raises mathematical questions. Statistical theory relies on probability and decision theory [5, 6].

A Fractal is a type of mathematical shape that are infinitely complex. In essence, a Fractal is a pattern that repeats forever, and every part of the Fractal, regardless of how zoomed in, or zoomed out you are, it looks very similar to the whole image. Fractals surround us in so many different aspects of life. Since the term is becoming more widely used we wanted to create the definitive guide to understanding what Fractals are, why Fractals are important, and how Fractals impact our lives. Before we begin exploring Fractals in detail, let's first take a look at some of the most common Fractals that you may encounter. The most commonly shown Fractal is called the Mandelbrot set, named after the mathematician Benoit Mandelbrot who coined the term Fractal [7]. The word Fractal comes from the Latin word Fractus, which means "broken" or "fractured," which is appropriate given that there are fractional components within each Fractal [1]. Fractals are the

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latest development in statistics. The space-time fluctuation pattern in dynamical systems was shown to have a self-similar or fractal structure in the 1970s (Mandelbrot, 1975). The larger scale fluctuation consists of smaller scale fluctuations identical in shape to the larger scale.

2. PRELIMINARIES

2.1 POPULATION

A statistical community is the dispose of all possible estimation on data communicate to the unbroken collection of portion for which an inference is to be made.

2.2 SAMPLE

A sample is part of the statistical population i.e. it is a subset which is collected to draw an inference about the population.

2.3 MEAN

The Mathematical mean using the formula.

$\overline{x} = \frac{x_1+x_2 \ldots\ldots x_n}{n}$

Example : If the weight of 5 people are 72kg, 86kg, 98kg, 56kg, 60kg. Find the mean weight of people.

$\overline{x} = \frac{72+86+98+56+60}{5}$

$= \frac{372}{5} = 74.4$

2.4 MEDIAN

The value of the middlemost observation obtain after arranging the data in ascending or descending order is it called median of data.

Example: 5, 5, 6, 1, 2 find the median for given data

Arrange the data in ascending order 1, 2, 5, 5, 6.

Thus the median = middle value = 5

2.5 VARIANCE

The variance is the square of standard deviation, i.e.,

Variance = (Standard deviation) ^{2}= σ^2 $\quad$ $S=\sqrt{\frac{\sum(x_i-\overline{x})}{n-1}}$

2.6 STANDARD ERROR

Mathematically proved that the standard error of $(\overline{x_1} - \overline{x_2})$ is $S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ where

$$S^2 = \frac{n_1 {s_1}^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

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2.7 t – TEST STATISTICS

A random sample of size n (n<30) with sample mean $\bar{x}$ and population standard deviation is not known for testing the population mean with specified value using two tail test or one tail test where

$t = \frac{\bar{x}-\mu}{S/\sqrt{n}}$ where $S^2 = \frac{n}{n-1}s^2$ where s is the standard deviation of sample. t - Distribution with number of degrees of freedom v = n-1.

2.8 STATISTICAL FRACTAL

Fractals are the latest development in Statistics. The space-time fluctuation pattern in dynamical systems was shown to have a self - similar or fractal structure in the 1970s (Mandelbrot, 1975). The larger scale fluctuation consists of smaller scale fluctuations identical in shape to the larger scale [7].

2.9 LACUNARITY

Lacunarity is another term due to Mandelbrot from Latin Lacuna which means gap. If a Fractal has large gaps or holes it has high lacunarity. Different fractals can be constructed that have the same dimension but that look widely different because they have different lacunarity [8].

$$L(r) = 1 + [\text{var}(r) / mean^2(r)]$$

3. THEOREM: *The components of totally disconnected space are its points*

Proof: It is sufficient to prove that any subspace of X with more than one point is disconnected. Let Y be a subspace of X with more than one point. Since X is totally disconnected if x, y ∈ Y if x ∉ y then x, y ∈ X and there exist a disconnection of x i.e.,

X = A∪B, A∩B = ∅ , A, B are open sets in X where x ∈ A, y ∈ B. Then Y = (A∩Y) ∪ (B∩Y) is a disconnection of Y.

That is Y is not connected. Hence any subspace of X having more than one point is disconnected.

Hence the component of totally disconnected space are its point [2, 3].

By applying this theorem in Statistical tool by assuming a Random sample of Post graduate students in a college. Considering their examination marks to know the efficiency of students as well as training given by the staff. Here X is taken as regular students and Y is taken as part time students. Both the components of X and Y are totally disconnected.

3.1 SAMPLING DISTRIBUTION

Here in this sampling distribution consider random sample of some Regular and Part time Student's marks to know their efficiency and significant level of training by using test statistics.

Consider an illustration: *The mark obtained by a group of 11 regular course students and another group of 11 part time course students in an examination is given below.*

Regular	*86*	*62*	*63*	*64*	*60*	*51*	*67*	*69*	*70*	*53*	*74*
Part- time	*62*	*70*	*71*	*62*	*60*	*56*	*75*	*64*	*72*	*58*	*66*

Examine whether the marks obtained by regular students and part time students differ significantly at 5% and 1% level of significance. So that Performance of students as well as faculty can be checked [4].

Solution: Letμ_1,μ_2 be the average marks by regular and part time students respectively.

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H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

Calculate the mean and variance

	d_1			d_2	
x	**x-66**	d_1^2	**y**	**y-65**	d_2^2
76	10	100	61	-4	16
62	-4	16	70	5	25
63	-3	9	71	6	36
65	-1	1	62	-3	9
63	-3	9	60	-5	25
64	-2	4	56	-9	81
68	2	4	75	10	100
69	3	9	64	-1	1
70	4	16	62	-3	9
54	-12	144	68	3	9
72	6	36	66	1	1
726	**0**	**348**	**715**	**0**	**312**

$\bar{x} = \frac{726}{11} = 66$, $\bar{y} = \frac{715}{11} = 65$

d_1 = x-66 d_2 = y-65

$$s_1^2 = \frac{\sum d_1^2}{n_1} - \left(\frac{\sum d_1}{n_1}\right)^2$$

$= \frac{348}{11} - 0$ $= \frac{348}{11}$

$$s_2^2 = \frac{\sum d_2^2}{n_2} - \left(\frac{\sum d_2}{n_2}\right)^2$$

$= \frac{312}{11} - 0$ $= \frac{312}{11}$

$$S^2 = \frac{n_1 {s_1}^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$= \frac{348+312}{11+11-2} = \frac{660}{21} = 31.428$

∴ S = 5.6060

The test statistic is

$t = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{66-65}{5.60\sqrt{\frac{1}{11}+\frac{1}{11}}} = \frac{1}{5.60*0.42}$

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$= \frac{1}{2.37} = 0.4219$

Number of degrees of freedom = $n_1 + n_2 - 2 = 20$

∴ Table value of t - statistics for 20 degree of freedom at 5% level of significance is 1.725

Inference:

H_0 is accepted at 5% level of significance since the calculated value of t is less than the table value of t.

Therefore by using the t test statistic the performance of both Regular student as well as part time students there is no significant difference in their examination due to the training given by the faculty in the reputed institution.

From the calculated mean and variance, lacunarity L(r) can be calculated

L(r) = 1 + [31.428 / 720.5]

= 1 + 0.0436

= 1.0436

This degree of point in Lacunarity implies that since both Regular and Part time course students has no significant difference in performance but all students have different marks [8].

4. RESULTS

A t-test is a statistical test that is used to compare the means of two groups. It is often used in hypothesis testing to determine whether a process or treatment actually has an effect on the population of interest, or whether two groups are different from one another. Here in this illustration the average performance of regular students μ_1 has no significant difference with part time students μ_2, using t-test the calculated value t = 0.4219 and the table value at 5% level of significance is 1.725 here calculated value is less than the table value so H_0 is accepted (i.e.) $\mu_1 = \mu_2$ performance of the Regular course students and Part time course students are equal this implies training given by the faculty has no significant difference. This sampling distribution can be compared with Statistical fractal i.e. the mean of two groups are irregular. Since this mark will not repeat in future examination there will be a fluctuation due to the reason like either regular or part time students not performing well in their exam or they won't get enough training in this case H_1 will be accepted. However there will be Self – Similarity [9] since both regular and part time course are identical in shape by Statistical fractals [10]. Therefore property of Fractals is also satisfied [1]. From the theorem both the X and Y are totally disconnected. Calculation of Lacunarity L(r) = 1.0436 measure the exact correspondence between different things which captures the degree to which points in a point set are totally disconnected from each other.

5. CONCLUSION

Use of statistical tools will visualise the complex data and explain lot of things within the short period of time. In this paper using t – test statistics analysed the performance of both students and faculty only with their marks this help us to improve their performance in future examination. The two mean groups satisfy self -Similarity of Fractals by Statistical Fractals. Lacunarity analysis is a multiscaled method for describing patterns of spatial dispersion.

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