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Inverse g-Eccentric Domination in Fuzzy Graph

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Abstract. A dominating set $D \subseteq V(G)$ in a fuzzy graph $G(\alpha,\beta)$ is said to be a g-eccentric dominating set if for every vertex b in $V - D$, $\exists$ at least one g-eccentric vertex a of b in D. If $V - D$ contains g-eccentric dominating set $D^{'}$ of a fuzzy graph $G(\alpha,\beta)$ then D' is called as an inverse g-eccentric dominating set with respect to D. The lowest cardinality taken over all the inverse g-eccentric dominating sets of G is called the inverse g-eccentric domination number. In this article, an inverse g-eccentric point set, the inverse g-eccentric dominating set and their numbers in fuzzy graphs are introduced. Bounds for some standard fuzzy graphs are obtained.

Keywords: g-Eccentric Dominating set, Inverse g-Eccentric Dominating set, Inverse g-eccentric domination number, Inverse g-eccentric point set, Inverse g-eccentric point number.

AMS Subject Classification 2020: 05C05, 05C12, 05C72.

I Introduction

The concept of fuzzy graphs(Simply written as FG) was pioneered by A. Rosenfeld [5] in the year 1975. V.R. Kulli and S.C. Sigarkanti developed the inverse domination in graph, in 1991[7]. In 2010, T.N. Janakiraman at.et., [3] began the eccentric domination in graph. In the year 2020, A.Mohamed Ismayil and S.Muthupandiyan [4] pioneered the g-eccentric domination in FG. R.Jahir Hussain and A. Fathima Begam [2] pioneered inverse eccentric domination in graphs in 2021.

The inverse g-eccentric point set, inverse g-eccentric dominating set, and their number in FG are presented in detail. For various standard FG, bounds for the inverse g-eccentric domination number are obtained. In this manuscript, some theorems on inverse g-eccentric domination in FG are posited and validated.

Harary [1] and A. Rosenfeld and S. Somasundaram [5, 6] might be used to refer to the graph and fuzzy graph theoretic terminologies, respectively.

Definition 1. [4, 5, 6] A FG $G = (\alpha,\beta)$ is characterized with two functions α on V and β on $E \subseteq V \times V$, where $\alpha : V \to [0,1]$ and $\beta : E \to [0,1]$ such that $\beta(a,b) \leq \alpha(a) \wedge \alpha(b), \forall a,b \in V$. We expect that V is a non-empty finite set, β is reflexive and symmetric functions. We indicate the crisp graph $G^* = (\alpha^*,\beta^*)$ of the FG $G(\alpha,\beta)$ where $\alpha^* = \{a \in V : \alpha(a) > 0\}$ and $\beta^* = \{(a,b) \in E : \beta(a,b) > 0\}$.

Definition 2. [4] A path P of length n is a sequence of distinct nodes $a_0, a_1, \ldots, a\_n$ such that $\mu(a_{i-1}, a_i) > 0, i = 1,2,..,n$ and strength of the path P is $s(P) = min\{\beta(a_{i-1}, a_i), i = 1,2,,\ldots n\}$.

Definition 3. [4] An edge is said to strong edges (or strong arc) if its weight is equal to the strength of connectedness of its end nodes. Symbolically, $\beta(a,b) \geq CONN_{G-(a,b)}(a,b)$.

Definition 4. [4, 6] The order and size of a FG $G(\alpha,\beta)$ are mentioned by $p = \sum_{a \in V} \tau(a)$ and $q = \sum_{ab \in E} \omega(a,b)$ respectively.

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Definition 5. [4] Let $G(\alpha, \beta)$ be a FG. The strong degree of a vertex $b \in \alpha^*$ is defined as the sum of membership values of all strong arcs incident at b and it is denoted by $d_s(b)$. It is also characterised by $d_s(b) = \sum_{\alpha \in N_s(b)} \beta(a, b)$ where $N_s(b)$ denotes the set of all strong neighbours of b.

Definition 6. [4] A strong path π from a to b is called geodesics in a FG in the event that there is no shorter strong path from a to b and a length of an $a - b$ geodesic is the geodesic distance(g-distance)from a to b and is intend through $d_g(a, b)$.

Note: The length of the geodesic distance $d_s(a, b)$ is the number of strong edges present in the path.

Definition 7. [3] The geodesic eccentricity (g-eccentricity) $e_g(a)$ of a node $a \in V$ in a connected FG $G = (\alpha, \beta)$ is characterized by $e_g(a) = max\{d_g(a, b), b \in V\}$. The least eccentricity among the vertices of G is g-radius and indicated by $r_g(G) = min\{e_g(a), a \in V\}$ and the greatest g-eccentricity among the vertices of G is called g-diameter and indicated by $d_g(G) = max\{e_g(a), a \in V\}$. A vertex b is a g-central vertex if $e_g(b) = r_g(G)$. Moreover, a vertex b in G is a g-peripheral vertex if $e_g(b) = d_g(G)$.

Definition 8. [4] Let $a, b \in V(G)$ be any two vertices in a FG $G(\alpha, \beta)$. A vertex a at g-distance $e_g(b)$ from b is a g-eccentric point of b. A vertex b's g-eccentric set is specified and intend through $E_g(b) = \{a / d_g(a, b) = e_g(b)\}$.

Definition 9. [4] The set $S \subseteq V$ in a FG $G(\alpha, \beta)$ is said to be a g-eccentric point set if for every $a \in V - S$, there exists at least one g-eccentric point b of a in S.

Definition 10. [4] A dominating set $D \subseteq V(G)$ in a FG $G = (\alpha, \beta)$ is said to be a g-eccentric dominating set if each vertex $b \notin D$, then there exists at least a g-eccentric vertex a of b in D. The least scalar cardinality taken over all gED-set is called gED-number and is intend through $\gamma_{ged}(G)$.

Definition 11. [2, 7] Let D be the lowest dominating set in a FG $G(\alpha, \beta)$. If $V - D$ contains a dominating set D' of G then D' is called an inverse dominating set related to D.

Definition 12. [4] A spider FG is a F-tree Sp_α, on $2n + 1$ vertices obtained by subdividing each edge of a F-star graph Sp_α, $|\alpha^*| = n + 1, n \geq 3$.

II Inverse g-Eccentric Point Set in a Fuzzy Graph

The inverse g-eccentric point set and its number in a FG $G(\alpha, \beta)$ are addressed in this topic.In this context, numerous observations have been made.

Definition 13. Let $S \subseteq V$ in a FG $G(\alpha, \beta)$ is a g-eccentric point set(gEP-set). If $V - S$ contains a g-eccentric point set S' of a FG $G(\alpha, \beta)$ then S' is called an inverse g-eccentric point set(INgEP-set) related to S. The inverse g-eccentric point set S' is minimal if there is no proper sub set S'' of S' in $G(\alpha, \beta)$. The lowest cardinality chosen over all the minimal INgEP-set is called the inverse g-eccentric number and is indicated by $e_g^{-1}(G)$ and simply intend through e_g^{-1}. The highest cardinality of a minimal INgEP-set is called an upper inverse g-eccentric point number and is intend through $E_g^{-1}(G)$ and simply intend through E_g^{-1}.

Example 1.

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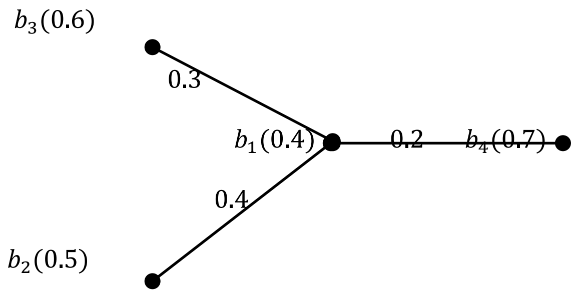

Figure 1

From the FG $G(\alpha,\beta)$ given in example 1, we observe that lowest g-EP-set is $S = \{b_2\}$. Then $V - S = \{b_1, b_3, b_4\}$ which contains g-EP-sets $S1 = \{b_3\}$ and $S_2 = \{b_4\}$. Therefore, S_1 is an INgEP-set and INgEP-number is $e_g^{-1}(G) = 0.6$ and S_2 is an upper inverse g-eccentric point set and upper INgEP-number is $E_g^{-1}(G) = 0.7$.

Observation 1:

(i) For any FG $G(\alpha,\beta), e_g(G) \leq e_g^{-1}(G) \leq E_g^{-1}(G)$.

(ii) If S_1 is an INgEP-set, then $S_1' \supset S_1$ is also an INgEP-set.

(iii) If S_1 is minimal INgEP-set , then$S_1' \subset S_1$ is not an INgEP- set.

(iv) In a F-tree T_α, every INgEP- set contains at least one pendent vertex.

(v) For any FG $G(\alpha,\beta)$, $e_g(G) + e_g^{-1}(G) \leq p$.

Observation 2 A path FG P_α , $|\alpha^*| = n, n \geq 4$ does not having INgEP-set.

Result 1

(i) Let K_α be any complete FG, the$e_g^{-1}(K_\alpha) = \alpha_0'$ where$|\alpha^*| = n, \alpha_0' = \min\{\alpha(a), a \in V - S\}$, where S is a lowest gEP-set of K_α.

(ii) Let $K_{\alpha_{1,\alpha_2}}$ be a complete bipartite FG, then $e_g^{-1}(K(\alpha_1,\alpha_2)) = \alpha_{10} + \alpha_{20}$, wher $|\alpha_1^*| = m$ and $|\alpha_1^*| = n, \alpha_{10} = \min\{\alpha(a), a \in V_1 - S\}, |\alpha_1^*| = n, \alpha_{20} = \min\{\alpha(b), b \in V_1 - S\},\ V2 - S\}, V = V_1 \cup V_2$.

(iii) Let S_α be a star FG. Then $e_g^{-1}(S_\alpha) \leq 1, |\alpha^*| = n, n \geq 3$.

(iv) Let W_α be a wheel FG. Then $e_g^{-1}(W_\alpha) \leq 2, |\alpha^*| = n, n \geq 5$.

(v) Let C_α be a cycle FG, $|\alpha^*| = n, n \geq 4$, then $e_g^{-1}(C_\alpha) = \begin{cases} \frac{P}{2}, n \text{ is even} \\ \frac{P-\alpha_0}{2}, n \text{ is odd} \end{cases}$

III Inverse g-Eccentric domination in a Fuzzy Graph

The inverse g-eccentric dominating set, and their number in several standard FG, are presented and analysed in this topic.

Definition 14. Let D be the lowest gED-set in a FG $G(\alpha,\beta)$. If $V - D$ contains a gED-set D' of a FG $G(\alpha,\beta)$ then D' is called an inverse g-eccentric dominating set(INgED-set) related to D. An INgED-set D' is called a minimal INgED-set if $D' \subset D$ is not an INgED-set. The lowest scalar cardinality of minimal INgED-set is known as the INgED-number of a FG G and is intend through $\gamma_{ged}^{-1}(G)$. The highest scalar cardinality of a minimal INgED-set is known as the upper INgED-number of a FG $G(\alpha,\beta)$ and is intend through $\Gamma_{ged}^{-1}(G)$.

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Example 2.

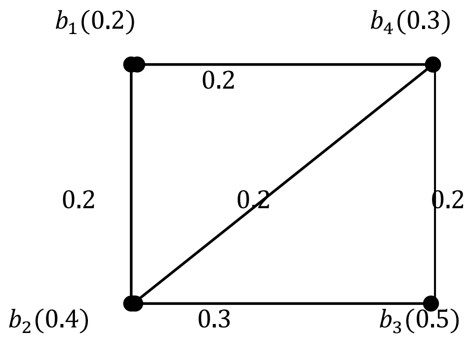


Figure 2

Consider the FG given in example 2, if a gED-set is $D = \{b_1, b_4\}$, then $V - D = \{b_2, b_3\}$. Consequently, $V - D$ have an INgED-set is $D' = \{b_2, b_3\}$. Hence, an INgED-number $\gamma_{ged}^{-1}(G)$ of a FG is 0.9. Also, upper INgED-number $\Gamma_{ged}^{-1}(G)$ of a FG is 0.9.

Observation 3

(i) If D is an INgED-set, then $D' \supset D$ is also an INgED-set.

(ii) If D is a minimal INgED-set, then $D' \subset D$ is not an INgED-set.

(iii) For any FG, if D is a lowest INgED-set then $V - D$ may or may not have any gED-set. So, INgED-set not exists for some FG.

Example, For a path FG $P_\alpha, |\alpha^*| = n, n = 3$, does not have an INgED-set.

Remark 1 Let D be an IND set in a FG $G(\alpha, \beta)$ and S be an INgEP-set of $G(\alpha, \beta)$. Then clearly $D \cup S$ is an INgED-set of $G(\alpha, \beta)$.

Remark 2 For any connected FG $G(\alpha, \beta)$

(i) $\gamma(G) \leq \gamma_{ged}(G) \leq \gamma_{ged}^{-1}(G)$.

(ii) $\gamma_{ged}^{-1}(G) \leq \Gamma_{ged}^{-1}(G)$.

(iii) $\gamma_{ged}(G) + \gamma_{ged}^{-1}(G) \leq p$

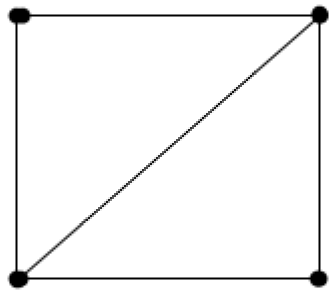
(v) For any star FG S_α, INgED-set does not exists.

Theorem 1. Let D be a lowest gED-set of a FG $G(\alpha, \beta)$. Then there exists an INgED-set D' of a FG $G(\alpha, \beta)$ in relation D if and only if every vertex in D has at least one g-eccentric vertex in $V - D'$.

Proof. Let D be a lowest gED-set of a FG $G(\alpha, \beta)$ and also G has INgED-set D' with respect to D. Let $D' = V - D$.

Then D' contains lowest gED-set. That's D' is also a gED-set. Therefore, every vertex in D has at least one g-eccentric vertex in $V - D'$. Conversely, let D be a lowest gED-set of a FG $G(\alpha, \beta)$ and every vertex in D has at least one g-eccentric vertex in $V - D$.

We know that $V - D$ is also a D-set of $G(\alpha, \beta)$. Therefore, $V - D$ is a gED-set of G with respect to D.



Consequently, $V - D$ contains lowest gED-set D'. Then D' is the lowest INgED-set with respect to D whose cardinality is the INgED-number of a FG $G(\alpha, \beta)$.

Theorem 2. $\gamma_{ged}^{-1}(K_\alpha) = \alpha_{10}, |\alpha^*| = n, n \geq 2$ where $\alpha_{10} = min\{\alpha(a), a \in$

$V - D\}$.

Proof. Let $b_1, b_2, \ldots, b\_n$ be the vertices of $K_\alpha, |\alpha^*| = n, n \geq 2$. We know that $\gamma_{ged}(K_\alpha) = \alpha_0$.

Let $D = \{b_1\}$ be the lowest gED-set of K_α.

Let any vertex b_i in $V - D$ is a lowest INgED-set. That is $D' = \{b_i \in V - \{b_1\}\}$ and anyone b_i in $D'$is a lowest INgED-set of a FG $G(\alpha, \beta)$ in relation D. Therefore, $\gamma_{ged}^{-1}(K_\alpha) = \alpha_{10}, |\alpha^*| = n, n \geq 2$ and $\alpha_{10} = min\{\alpha(a), a \in V - D\}$.

Theorem 3. For a complete bipartite FG, $\gamma_{ged}^{-1}(K_{\alpha_{1,\alpha_2}}) = \alpha_{10} + \alpha_{20}$, where $\alpha_{10} = \min_{a \in V_1} \alpha(a)$ and $\alpha_{20} = \min_{b \in V_2} \alpha(b)$.

Proof. $(K_{\alpha_{1,\alpha_2}}), \alpha = \alpha 1 \cup \alpha 2$ be any bipartite complete FG, where $|\alpha_1^*| = m$, $|\alpha_2^*| = n$,and $m, n \geq 2$ then each point a of V_1 is adjacent to every point b of V_2 and vice versa.

Let $D = \{a_1, b_1\}$ where $a_1 = min\{\alpha(a), a \in V_1\}$ and $b_1 = min\{\alpha(b), b \in V_2\}$ is a lowest gED-set. Then any vertex $a_i \in V_1 - \{a_1\}$ dominates all the vertices of V_2 and it is a g-eccentric vertex of all the vertices in $V_1 - \{a_i\}$. Similarly for $V_2 - \{b_1\}$.

Let $D' = \{a_i, b_i\}$ where $a_i \in V_1 - \{a_1\}, i \neq 1$ and $b_i \in V_2 - \{b_1, i \neq 1\}$. Then any two vertices $\{a_i, b_i\} \subseteq D'$ where $a_i \in V_1 - \{a_1\}$ and $b_i \in V_2 - \{b_1\}$ is a lowest INgED-set of a FG $G(\alpha, \beta)$ in relation D. Consequently, $\gamma_{ged}^{-1}(K_{\alpha_{1,\alpha_2}}) = \alpha_{10} + \alpha_{20}$, where $\alpha_{10} = \min_{a \in V_1} \alpha(a)$ and $\alpha_{20} = \min_{b \in V_2} \alpha(b)$, $|\alpha_1^*| = m$, $|\alpha_2^*| = n$,and $m, n \geq 2$.

Theorem 4. Let W_α be wheel FG, $|\alpha^*| = n$ then the following results holds

(i) $\gamma_{ged}^{-1}(W_\alpha) = \alpha_{10}, |\alpha^*| = 4$

(ii) $\gamma_{ged}^{-1}(W_\alpha) \leq 2, |\alpha^*| = 5$

(iii) $\gamma_{ged}^{-1}(W_\alpha) \leq 3, |\alpha^*| = 6$

(iv) $\gamma_{ged}^{-1}(W_\alpha) \leq 2, |\alpha^*| = 7$

(v) $$\gamma_{ged}^{-1}(W_\alpha) \leq \begin{cases} \frac{p-1}{3}, |\alpha^*| = n, n \geq 8 \; and \; n = 3m + 1, m \geq 3 \\ \frac{p}{3}, |\alpha^*| = n, n \geq 8 \; and \; n = 3m \; or \; n = 3m + 1, m \geq 3 \end{cases}$$

Proof:

(i) Let $G = W_\alpha$ and $|\alpha^*| = 4$. We know that $W_\alpha = K_\alpha$. Hence by a theorem 2 $\gamma_{ged}^{-1}(W_\alpha) = \alpha_{10}$.

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(ii) Let $G = W_\alpha$ and $|\alpha^*| = 5$. We know that $\gamma_{ged}(W_\alpha) \leq 2$. Let $D = \{a_1, b_2\}$ is a lowest gED-set. Then $V - D = \{a_3, a_4, b\}$, where b is the central vertex of $W_{\alpha.}$ Consider $D' = \{a_3, a_4\} \subseteq V - D$ which is a lowest INgED-set of G with respect to D. Consequently, $\gamma_{ged}^{-1}(W_\alpha) \leq 2, |\alpha^*| = 5$

(iii) Let $G = W_\alpha$ and $|\alpha^*| = 6$. Let $D = \{a_1, a_2, b\}$ is a lowest gED-set, where a_1, a_2 are adjacent non-central vertices and b is the central vertex. Consider $' = V - D = \{a_3, a_4, a_5\}$, which is the lowest IngED- set. Therefore, $\gamma_{ged}(W_\alpha) \leq 3, |\alpha^*| = 6$.

(iv) Let $G = W_\alpha$ and $|\alpha^*| = 7$. Let $D = \{a_1, a_4\}$ is a lowest gED-set. Then $V - D = \{a_2, a_3, a_5, a_6, b\}$ where b is the central vertex. Consider $D' = \{a_2, a_5\}$ where a_2 dominates a_1, a_3, v and a_5 dominates a_6, a_4, b and also a_2 is an gE-point of a_6, a_4, b and a_5 is an gE-point of a_1, a_3, b. Therefore, D' is the lowest IngED-set with respect to D. Hence, $\gamma_{ged}^{-1}(W_\alpha) \leq 2, |\alpha^*| = 7$.

(v) Let $G = W_\alpha$ and $|\alpha^*| = n, n \geq 8$. Let $D = \{a_1, a_2\}$ is a lowest gED-set of G where b is the central vertex and a_1, a_2 are adjacent non-central vertices. Then $V - D = \{a_3, a_4, \cdots, a_{n-1}\}$. In W_α each vertex a_i dominates two adjacent non-central vertices a_{i-1}, a_{i+1} and the central vertex b and also each non-central vertices is the g-eccentric vertex of all other non-adjacent non-central vertices and adjacent central vertex.

Case (i) n is even

(a) If $n = 2k = 3m + 1$ **(m is odd)**

$\Rightarrow 2k = 3(m-1) + 4$

$\Rightarrow k = \frac{3(m-1)}{2} + 2$

$\Rightarrow k = 3l + 2 \left[Since\ let \frac{(m-1)}{2} = l\right]$

Consider, $D' = \{a_3, a_6, \cdots, a_{k-2}, a_{k+1}, a_{k+4}, \cdots, a_{2k-4}, a_{2k-1}\}$ is a minimum INgED-set of G with respect to D.

$\gamma_{ged}^{-1}(W_\alpha) \leq \frac{p-1}{3}$, $|\alpha^*| = n, n = 2k = 3k + 1$ ... (1)

(b) If $n = 2k = 3m$ (m is even)

$\Rightarrow k = 3l$ [$since\ let\ l = \frac{m}{2}$]

Consider, $D" = \{a_3, a_6, \ldots a_{k-3}, a_k, a_{k+3}, \ldots, a_{2k-3,} a_{2k-1}\}$ is a lowest INgEDset of G related to D.

$\gamma_{ged}^{-1}(W_\alpha) \leq \frac{p}{3}$, $|\alpha^*| = n, n = 2k = 3m$ ... (2)

(c) If $n = 2k = 3m + 2$ **(m is even)**

$\Rightarrow k = 3\ \frac{m}{2} + 1$

$\Rightarrow k = 2l + 1[since\ let \frac{m}{2} = l]$

Consider, $D' = \{a_3, a_6, \ldots, a_{k-1}, a_{k+2}, a_{k+5}, \ldots, a_{2k-2}\}$ is a lowest INgEDset of G related to D. Therefore, $\gamma_{ged}^{-1}(W_\alpha) \leq \frac{p}{3}$, $|\alpha^*| = n, n = 2k = 3k + 2$ ... (3)

From (1),(2) and (3),

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$$\gamma_{ged}^{-1}(W_\alpha) \le \begin{cases} \frac{p-1}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m+1, m \ge 3 \\ \frac{p}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m \; or \; n = 3m+1, m \ge 3 \end{cases} \dots (4)$$

Case (ii) n **is odd**

(a)If $n = 2k + 1 = 3m + 1$ (m ***is even***)

$\Rightarrow 2k = 3m$

$\Rightarrow k = 3l$ [$since\ let\ l = \frac{m}{2}$]

Consider, $D' = \{a_3, a_6, \dots, a_{k-3}, a_k, a_{k+3}, \dots, a_{2k-3}, a_{2k}\}$ is a INgED-set of G related to D. $\gamma_{ged}^{-1}(W_\alpha) \le \frac{p-1}{3}$, $|\alpha^*| = n, n = 2k+1 = 3m+1 \dots (5)$

(b)If $n = 2k + 1 = 3m$ (m ***is odd***)

$\Rightarrow 2k = 3(m-1) + 2$

$\Rightarrow k = 3\frac{m-1}{2} + 1$

$\Rightarrow k = 3l + 1$ [$since\ let\ l = \frac{m-1}{2}$]

Consider, $D' = \{a_3, a_6, \cdots, a_{k-4}, a_{k-1}, a_{k+2}, \dots, a_{2k-5}, a_{2k-2}, a_{2k}\}$ is a lowest INgED-set of G related to D. $\gamma_{ged}^{-1}(W_\alpha) \le \frac{p}{3}$, $|\alpha^*| = n, n = 2k+1 = 3m \dots (6)$.

(c)If $n = 2k + 1 = 3m + 2$ (m ***is odd***)

$\Rightarrow 2k = 3(m-1) + 4$

$\Rightarrow k = 3\frac{m-1}{2} + 2$

$\Rightarrow k = 3l + 2$ [$since\ let\ l = \frac{m-1}{2}$]

Consider, $D' = \{a_3, a_6, \cdots, a_{k-2}, a_{k+1}, a_{k+4}, \dots, a_{2k-4}, a_{2k-1}, a_{2k}\}$ is a lowest INgED-set of G related to D. $\gamma_{ged}^{-1}(W_\alpha) \le \frac{p}{3}$, $|\alpha^*| = n, n = 2k+1 = 3m+2 \dots (7)$.

From (5), (6) and (7)

$$\gamma_{ged}^{-1}(W_\alpha) \le \begin{cases} \frac{p-1}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m+1, m \ge 3 \\ \frac{p}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m \; or \; n = 3m+1, m \ge 3 \end{cases} \dots (8)$$

Hence from (4) and (8)

$$\gamma_{ged}^{-1}(W_\alpha) \le \begin{cases} \frac{p-1}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m+1, m \ge 3 \\ \frac{p}{3}, |\alpha^*| = n, n \ge 8 \; and \; n = 3m \; or \; n = 3m+1, m \ge 3 \end{cases} \dots (4).$$

IV Bounds on Inverse g-Eccentric Domination in Fuzzy Graph

In this section bounds on INgED in FG are discussed.

Theorem 5. For a spider FG $Sp_\alpha, \gamma_{ged}^{-1}(Sp_\alpha) \le p - \Delta_s(Sp) - 1$ where $|\alpha^*| = n, n = 2k + 1 \ge 9$ is the number of vertices of Sp_α.

Proof. Let Sp_α be a spider FG, $|\alpha^*| = n, n = 2k+1 \geq 9$. Then $r_g(Sp) = 2$ and $d_g(Sp) = 4$.

We know that $\gamma_{ged}(Sp_\alpha) \leq p - \Delta_s(Sp) - 1 = k = |N_s(a)|$ where a is the central vertex of Sp_α.

Let D be a lowest GED-set containing $k - 2$ vertices of $N_s(a)$ and 2 pendent vertices of Sp_α which are not adjacent to the vertex which we have selected from $N_s(a)$ to D. Then $D' = V - D$ contains remaining $k - 2$ pendent vertices, and 2 vertices from $N_s(a)$ and the central vertex a.

Let D'' be the subset of D' containing $k - 2$ pendent vertices and 2 vertices of $N_s(a)$. Then D'' is the lowest INgED-set of Sp_α with respect to D. Therefore, $\gamma_{ged}^{-1}(Sp_\alpha) = |D''| = k - 2 + 2 = k = p - \Delta_s(Sp_\alpha) - 1$.

Theorem 6. For any connected FG, $G(\alpha,\beta), \gamma_{ged}^{-1}(G) \leq \gamma^{-1}(G) \leq e_g^{-1}(G)$.

Proof: By Remark 1, every INgED-set is the union of inverse dominating set and INgEP-set. Hence, , $G(\alpha,\beta), \gamma_{ged}^{-1}(G) \leq \gamma^{-1}(G) \leq e_g^{-1}(G)$.

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