JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1982-1984
https://publishoa.com
ISSN: 1309-3452

$[\mathbf{1,2}]$-Complementary connected domination number for total graphs

Renuka Kolandasamy

Assistant Professor, Department of Engineering Mathematics, College of Engineering,

Koneru Lakshmaiah Education Foundation, Vaddeswaram, AP, India

Email ID: math.renuka@kluniversity.in

Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

ABSTRACT

A set $S \subseteq V(G)$ in a graph G is said to be a [1,2]-complementary connected dominating set, if for every vertex $v \in V - S$, $1 \leq |N(v) \cap S| \leq 2$ and $< V - S >$ is connected. The minimum cardinality of a [1,2]-complementary connected dominating set is called the [1,2]-complementary connected domination number and is denoted by $\gamma_{[1,2]cc}(G)$. In this paper, we exhibited the results based on [1,2]-complementary connected domination number for total graph.

Keywords: Complementary connected domination, [1,2]-domination, [1,2]-complementary connected domination

AMS Subject Classification: $05C69$

1 Introduction

The graph $G = (V, E)$, we mean a finite, undirected, connected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak [3] and Haynes et.al [4].

V. R. Kulli and B. Janakiraman [5] introduced the concept of nonsplit domination in graphs. Later Tamilchelvam [8] introduced the same concept in different name as complementary connected domination in graphs. Mustapha Chellali et.al., [7] first studied the concept of [1,2]-sets. Xiaojing Yang and Baoyindureng Wu [13] extended the study of this parameter. In [9], K. Renuka, et.al., introduced the concept of [1,2]-complementary connected domination number of graphs and studied its character and in [8], K. Renuka et.al., studied about cubic graphs in [1,2]-complementary connected domination number of graphs. In [11], T.Tamizh Chelvam et.al., studied the concept of complementary connectedness of graphs. In [12], J.Vernold Vivin et.al., studied about on harmonous coloring of total garphs of cycle, path and star graphs.

In [1], M. M. Akbar Ali, et.al., discuseed about equitable coloring of central and total graphs in 2009. Later in [2], M. M. Akbar Ali and S. Panayappan discusses about cycle multiplicity of total graphs of cycle, path and star graphs in 2010. In [9], J. Vernold Vivin, et.al., discussed the results about harmonous coloring of total graphs for various graphs. In [6], T. P. Latchoumi et.al., studies about enhanncement system using grey-fuzzy graph. In [10], TL Yookesh et.al, studied defuzzificztion formula for modelling and scheduling for fuzzy project network.

Motivated by the above concepts, in this paper we found [1,2]-complementary connected domination number for total graphs.

2 Main Result

Theorem 2.1 $$\boldsymbol{\gamma_{[1,2]cc}(T(P_n))} = \begin{cases} \left\lfloor \frac{2n-1}{5} \right\rfloor & \boldsymbol{n \equiv 2 (mod\ 3), for\ any\ n \geq 11} \\ \left\lceil \frac{2n-1}{5} \right\rceil & \boldsymbol{n \equiv 2 (mod\ 3), for\ any\ n \leq 8} \\ \left\lceil \frac{2n-1}{5} \right\rceil & \boldsymbol{n \equiv 0, 1 (mod\ 3)} \end{cases}$$

Proof. Let v_i be the vertices of P_n, where $1 \leq i \leq n$ and $v_{i'}$ be the corresponding vertices of edge $v_i v_{i+1}$. Let

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1982-1984
https://publishoa.com
ISSN: 1309-3452

$V[T(P_n)] = \{v_i, v_{i'}\}$ and $|V[T(P_n)]| = 2n-1$. If $n=2$, then $|V[T(P_n)]| = 3$, so that $\{v_1, v_{1'}, v_2\}$ be the vertices of $V[T(P_2)]$. Now, $v_{1'}$ forms the $[1,2]cc$-set and hence $\gamma_{[1,2]cc}(T(P_2)) = 1$. If $n > 2$, then $|V[T(P_n)]| = 2n-1$, so that $\{v_1, v_{1'}, v_2, v_{2'}, \dots, v_{n-1}, v_{n-1}', v_n\}$ be the vertices of $V[T(P_n)]$.

Case 1: $n \equiv 2(mod\ 3)$

The set $S_1 = \{v_2, v_7, v_{4'}\}$ forms $[1,2]cc$-set of $T(P_n)$, for any $n \le 8$. Hence $\gamma_{[1,2]cc}(T(P_n)) = \left\lceil \frac{2n-1}{5} \right\rceil$. The set $S_2 = \{v_2, v_7, v_{12}, \dots, v_n, v_{4'}, v_{9'}, \dots, v_{n-2}'\}$ forms $[1,2]cc$-set of $T(P_n)$, for any $n \ge 11$. Hence $\gamma_{[1,2]cc}(T(P_n)) = \left\lfloor \frac{2n-1}{5} \right\rfloor$.

Case 2: $n \equiv 0,1(mod\ 3)$

The sets $S_1 = \{v_2, v_7, v_{12}, \dots, v_n, v_{4'}, v_{9'}, \dots, v_{n-5}'\}$ form $[1,2]cc$-set of $T(P_n)$, for any $n \equiv 0(mod\ 3)$ and $S_2 = \{v_2, v_7, v_{12}, \dots, v_{n-4}, v_{4'}, v_{9'}, \dots, v_{n-1}'\}$ form $[1,2]cc$-set of $T(P_n)$, for any $n \equiv 1(mod\ 3)$. Hence $\gamma_{[1,2]cc}(T(P_n)) = \left\lceil \frac{2n-1}{5} \right\rceil$.

Theorem 2.2 $\gamma_{[1,2]cc}(T(K_{1,n-1})) = 1, \textit{for any } n \ge 2$

Proof. Let v_0 be central vertex of star graph and $S_1 = \{v_1, v_2, \dots, v_{n-1}\}$ be the vertices of pendant in star graph. Let $S_2 = \{v_{1'}, v_{2'}, \dots, v_{n-1}'\}$ be the vertices of $T(K_{1,n-1})$. Since v_0 is adjacent to both the set S_1 and S_2, which forms $[1,2]cc$-set and hence $\gamma_{[1,2]cc}(T(K_{1,n-1})) = 1$.

Theorem 2.3 $\gamma_{[1,2]cc}(T(C_n)) = \begin{cases} \left\lfloor \frac{2n-1}{5} \right\rfloor & n \equiv 2(mod\ 3), for\ any\ n \ge 11 \\ \left\lceil \frac{2n-1}{5} \right\rceil & n \equiv 2(mod\ 3), for\ any\ n \le 8 \\ \left\lceil \frac{2n-1}{5} \right\rceil & n \equiv 0,1(mod\ 3) \end{cases}$

Proof. Let v_i be the vertices of C_n, where $1 \le i \le n$ and $v_{i'}$ be the corresponding vertices of edge $v_i v_{i+1}$. Let $V[T(C_n)] = \{v_i, v_{i'}\}$ and $|V[T(C_n)]| = 2n-1$. If $n=2$, then $|V[T(C_n)]| = 3$, so that $\{v_1, v_{1'}, v_2\}$ be the vertices of $V[T(C_2)]$. Now, $v_{1'}$ forms the $[1,2]cc$-set and hence $\gamma_{[1,2]cc}(T(C_2)) = 1$. If $n > 2$, then $|V[T(C_n)]| = 2n-1$, so that $\{v_1, v_{1'}, v_2, v_{2'}, \dots, v_{n-1}, v_{n-1}', v_n\}$ be the vertices of $V[T(C_n)]$.

Case 1: $n \equiv 2(mod\ 3)$

The set $S_1 = \{v_2, v_7, v_{4'}\}$ forms $[1,2]cc$-set of $T(C_n)$, for any $n \le 8$. Hence $\gamma_{[1,2]cc}(T(C_n)) = \left\lceil \frac{2n-1}{5} \right\rceil$. The set $S_2 = \{v_2, v_7, v_{12}, \dots, v_n, v_{4'}, v_{9'}, \dots, v_{n-2}'\}$ forms $[1,2]cc$-set of $T(C_n)$, for any $n \ge 11$. Hence $\gamma_{[1,2]cc}(T(C_n)) = \left\lfloor \frac{2n-1}{5} \right\rfloor$.

Case 2: $n \equiv 0,1(mod\ 3)$

The sets $S_1 = \{v_2, v_7, v_{12}, \dots, v_n, v_{4'}, v_{9'}, \dots, v_{n-5}'\}$ form $[1,2]cc$-set of $T(P_n)$, for any $n \equiv 0(mod\ 3)$ and $S_2 = \{v_2, v_7, v_{12}, \dots, v_{n-4}, v_{4'}, v_{9'}, \dots, v_{n-1}'\}$ form $[1,2]cc$-set of $T(C_n)$, for any $n \equiv 1(mod\ 3)$. Hence $\gamma_{[1,2]cc}(T(C_n)) = \left\lceil \frac{2n-1}{5} \right\rceil$.

Theorem 2.4 $\gamma_{[1,2]cc}(W_n) = 1 + \left\lceil \frac{n-1}{3} \right\rceil, \textit{for any } n \ge 4.$

Proof. Let $(v_1, v_2, \dots, v_n)$ be the vertices of W_n and v_1 be the central vertex of W_n and $\{v_2, v_3, \dots, v_n\}$ be the outer vertices of W_n. Let $\{v_1, v_2, \dots, v_n, v_{2'}, v_{3'}, v_{4'}, \dots, v_{n'}, u_{2'}, u_{3'}, \dots, u_{n'}\}$ be the vertices of $T(P_n)$. If $n = 4$, then $\{v_1, v_{2'}\}$ forms $[1,2]cc$ -set and $\gamma_{[1,2]cc}(W_4) = 2$. Here, $S_1 = \{v_1, v_{2'}, v_{5'}, v_{8'}, \dots, v_{n-1}'\}$ forms $[1,2]cc$ -set when $n \equiv 0, 2(mod\ 3)$ and $S_2 = \{v_1, v_{2'}, v_{5'}, v_{8'}, \dots, v_{n-2}'\}$ forms $[1,2]cc$-set when $n \equiv 1(mod\ 3)$. Hence, $\gamma_{[1,2]cc}(W_n) = 1 + \left\lceil\frac{n-1}{3}\right\rceil$.

Theorem 2.5 $\gamma_{[1,2]cc}(T(F_r)) = r + 1$, *for any* $1 \le r \le n$ *and* $n \le 2$.

Proof. Let $V(F_r) = (v_1, v_2, \dots, v_n)$. F_r is constructed by r copies of cycle C_3 with common vertex and v_1 is the central vertex of F_r and $\{v_2v_3, v_4v_5, \dots, v_{n-1}v_n\}$ be the wings of F_r. Let $\{v_{1'}, v_{2'}, v_{3'}, v_{1''}, v_{2''}, v_{3''}, \dots, v_1^r, v_2^r, v_3^r\}$, where $1 \le r \le n$ be the vertices corresponding to the edges $E(F_r)$. $V(T(F_r)) = \{v_i : 1 \le i \le n\} \cup \{v_{i'}, v_{i''}, \dots, v_i^r : 1 \le i \le 3\ and\ 1 \le r \le n\}$. Let $S = \{v_1, v_i^r : i = 3, 1 \le r \le n\}$, where $i \le r \le n$ form $[1,2]cc$-set and $|S| = r + 1$. Hence $\gamma_{[1,2]cc}(T(F_r)) = r + 1$.

References

1. M. M. Akbar Ali, K. Kaliraj and J. Vernold Vivin, On Equitable coloring of central graphs and total graphs, Electronic notes in discrete mathematics, 33, (2009), 1-6.
2. M. M. Akbar Ali and S. Panayappan, Cycle multiplicity of total graphs of C_n, P_n and $K_{1,n}$, International journal of engineering, science and techonology, 2(2), (2010), 54-58.
3. G. Chartrand and L. Lesniak, Graphs and Digraphs, Fourth Edition CRC Press, Boca Raton, 2005.
4. T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1998.
5. V. R. Kulli and B. Janakiraman, The nonsplit domination number of a graph, Indian journal of pure and applied mathematics, 31(5), (2000), 545-550.
6. T. P. Latchoumi, G. Kalusuraman, J. F. Banu, T. L. Yookesh, T. P. Ezhilarasi and K. Balamurugan, "Enhancement in manufacturing systems using Grey-Fuzzy and LK-SVM approach," 2021 IEEE International Conference on Intelligent Systems, Smart and Green Technologies (ICISSGT), doi: 10.1109/ICISSGT52025.2021.00026, (2021), 72-78.
7. Mustapha Chellali, Teresa W. Haynes, Stephen T. Hedetniemi and Alice McRae, [1,2]-Sets in graphs, Discrete Applied Mathematics, 161, (2013), 2885-2893.
8. G. Mahadevan and K. Renuka, [1,2]-Complementary connected domination in graphs-III, Communications Faculty Science University of Ankara Series A1 Mathematics and Statistics, 68(2), (2019), 2298-2312.
9. G. Mahadevan, K. Renuka and C. Sivagnanam, [1,2]-Complementary connected domination in graphs, International Journal of Computational and Applied Mathematics, 12(1), (2017), 281 − 288.
10. S Maheswari, M Shalini, TL Yookesh, Defuzzification formula for modelling and scheduling a furniture fuzzy project network Int. J. Eng. Adv. Technol. 9 (5), (2019), 279-283.
11. T. Tamizh Chelvam and B. JayaPrasad, Complementary connected domination number, International journal of management systems, 18(2), (2002), 149 − 152.
12. J. Vernold Vivin, M. M. Akbar Ali and K. Kaliraj, On harmonous coloring of total graphs of $C(C_n)$, $C(K_{1,n})$ and $C(P_n)$, Proyecciones journal of mathematics, 29(1),(2010), 57-76.
13. Xiaojing Yang and Baoyindureng Wu, [1,2]-domination in graphs, Discrete Applied Mathematics, 175, (2014), 79-86.

.