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Inverse Independent Domination in Interval Graphs

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Abstract

A set $I_D \subseteq V(G)$ is an independent dominating set if $\forall\ x, x \in V - I_D$ is adjacent to at least one vertex in I_D and no two vertices in I_D are adjacent. A set I'_D is called an inverse independent dominating set w.r.t I_D if $V - I_D$ contains an independent dominating set, where I_D is a minimum independent dominating set of graph $G = (V, E)$. In this paper, we develop an algorithm for existence of an inverse independent dominating set w.r.t a minimum independent dominating set for an interval family and also, we obtain some results.

Keywords: Minimum independent dominating set, Interval family, Inverse independent dominating set and Inverse independent domination number.

1. Introduction

Here, we have considered the graphs which are undirected, finite, non-trivial, simple and having no isolated vertices. In 1962, Ore [9] and Berge[8] characterized the theory of independent domination. Further, Hedetniemi and Cockayne in [10] proposed the notation $i(G)$ which represents an independent domination number.

In the year 1991, V.R.Kulli et.al [2] proposed the idea of inverse domination and defined the following definition:

A set D'_s is called an inverse dominating set w.r.t D_s if $V - D_s$ contains a dominating set, where D_s is a minimum dominating set of graph G.

Further, in the year 2012, V.R.Kulli et.al [6] proposed the idea of inverse independent domination as follows:

A set I'_D is called an inverse independent dominating set w.r.t I_D if $V - I_D$ contains an independent dominating set, where I_D is a minimum independent dominating set of graph G. The minimum number of elements in the set I'_D is an inverse independent domination number and it is denoted as $i'(G)$.

Every graph has an inverse independent dominating set $I_D{}'$ if it contains no isolated vertices. If no two vertices in a graph G are adjacent, then the collection is said to be an independent set. An independence number is the number of vertices in the largest independent set, and it is denoted by $\alpha(G)$. We discuss the correlation between $\alpha(G)$ and $i'(G)$ of a simple and finite connected graphs.

In 2020, Dr. V Ramalatha and Purushotham P [4] developed an algorithm for the existence of an inverse dominating set of an interval family. Motivated by this algorithm, we develop an inverse independent dominating set algorithm w.r.t a minimum independent dominating set for the same family. Many other authors have studied the concept of an inverse domination and an inverse independent domination in domination theory, for example [1, 5, 7, 11, 12, 13, 14, 17, 18].

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Let $I=\{i_1,i_2,.....,i_n\}$ be an interval family. Each interval i_j in I, for $i=1,2,...,n$ and $j=1,2,...,n$ is expressed by $[a_i,b_i]$, where a_i is left side end point and b_i is right side end point of I. If there is a 1-1 correspondence between V and I in such a way that two vertices of graph G are connected by an edge E provided their respective intervals in the interval family I intersects, then the graph G is called an interval graph.

2. Observations

2.1:[6] *For a cycle C_p with p vertices,* $i'(C_p)=\left\lceil \frac{p}{3} \right\rceil$.

2.2:[6] *For a path P_p with p vertices,* $i'(P_p)=\left\lceil \frac{p}{3} \right\rceil+1$ *if* $p\equiv 0(mod\ 3)$

$$=\left\lceil \frac{p}{3} \right\rceil \text{otherwise.}$$

2.3:[6] *For a wheel W_p with p vertices, where $p\geq 4$ then* $i'(W_p)=\left\lceil \frac{p-1}{3} \right\rceil$.

2.4:[6] *For a complete graph K_p with p vertices* $i'(K_p)=1$.

2.5:[6] *For a complete bipartite graph $K_{m,n}$ with p vertices,* $i'(K_{m,n})=p-1$ *where* $p=m+n$ *if* $m=1$ *or* $n=1,2$

3. Fundamental results

3.1:[6] *In G, if there are no isolated vertices then* $i(G)\leq i'(G)$. *Furthermore, the equality holds if* $G=K_p$, $p\geq 2$; C_p, $p\geq 3$; P_{3k+1} *or* P_{3k+2}, $k\geq 1$.

3.2:[6] *In G, if there are no isolated vertices then* $i(G)+i'(G)\leq p$. *Furthermore, the equality holds if* $G=K_2,P_3,P_4$ *or* C_4.

4. Algorithm

Some Notations

1. M_{ID} = Minimum independent dominating set.
2. I_{ID} = Inverse independent dominating set.
3. $Nbd^+[x]$ = The collection of all intervals in the right end side intersecting interval to the interval $'x'$.
4. $Max(S)$ = The largest number in a set S.
5. $NI^+(a)$ = First non-intersecting right end side interval of an interval $'a'$.

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Algorithm 1: Inverse independent dominating set algorithm for an interval family w.r.t minimum independent dominating set.

INPUT: Let $I=\{1,2,......,n\}$ be a given interval family.

Step $_1$ **:** $M_{ID}=\{\}$

Step $_2$ **:** $I_{ID}=\{\}$

Step $_3$ **:** S= $Nbd^{+}[1]$

Step $_4$ **:** S_1 =The set of intervals which are intersecting to all other intervals in S.

Step $_5$ **:** a=max(S_1)

Step $_6$ **:** $M_{ID}=M_{ID}\cup\{a\}$

Step $_7$ **:** $S_2=S_1-\{a\}$

Step $_8$ **:** If S_2 is null then

Step $_{8.1}$ **:** b=first right intersection of $'a'$.

Else

Step $_{8.2}$ **:** b=max(S_2)

Step $_9$ **:** $I_{ID}=I_{ID}\cup\{b\}$

Step $_{10}$ **:** x= $NI^{+}(a)$

Step $_{11}$ **:** If x is not null then

Step $_{11.1}$ **:** $SS_1=Nbd^{+}[x]$

Step $_{11.2}$ **:** SS_{11} = The set of intervals which are intersecting to all other intervals in SS_1.

Step $_{11.3}$ **:** a=max(SS_{11})

Step $_{11.4}$ **:** If a $\in$ Nbd(i) for any i $\in M_{ID}$ then

Step $_{11.4.1}$ **:** $SS_{11}=SS_{11}-\{a\}$

Step $_{11.4.2}$ **:** a=max(SS_{11}) goto step 11.4

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Step $_{11.5}$ **:** $M_{ID} = M_{ID} \cup \{a\}$

Step $_{12}$ **:** y= $NI^{+}(b)$

Step $_{13}$ **:** If y is not null then

Step $_{13.1}$ **:** $SS_2 = Nbd^{+}[y]$

Step $_{13.2}$ **:** SS_{22} = The set of intervals which are intersecting to all other intervals in SS_2 .

Step $_{13.3}$ **:** b=max(SS_{22})

Step $_{13.4}$ **:** If $b \in M_{ID}$ then

Step $_{13.4.1}$ **:** $SS_{22} = SS_{22} - \{b\}$

Step $_{13.4.2}$ **:** b=max(SS_{22}) goto step 13.4.

Step $_{13.5}$ **:** $I_{ID} = I_{ID} \cup \{b\}$ goto step 10.

Step $_{14}$ **:** End

OUTPUT: I_{ID} is the required inverse independent dominating set w.r.t a minimum independent dominating set M_{ID}

Illustration 1:

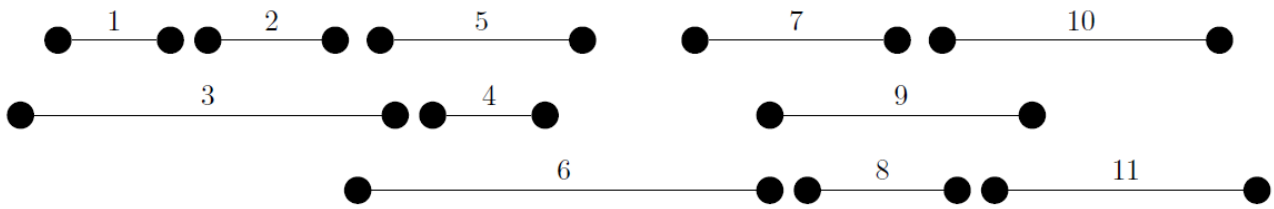

Figure: Interval family

Algorithm 1: Illustration for the algorithm 1

INPUT: Let $I = \{1,2,......,11\}$ be a given interval family.

Step $_{1}$ **:** $M_{ID} = \{\}$

Step $_{2}$ **:** $I_{ID} = \{\}$

Step $_{3}$ **:** S= $\{1,3\}$

Step $_{4}$ **:** $S_1 = \{1,3\}$

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Step $_5$ **:** a=3

Step $_6$ **:** M_{ID} ={3}

Step $_7$ **:** S_2 ={1}

Step $_{8.2}$ **:** b=1

Step $_9$ **:** I_{ID} ={1}

Step $_{10}$ **:** x=4

Step $_{11.2}$ **:** SS_{11}={4,5,6}

Step $_{11.4.2}$ **:** a=4

Step $_{11.5}$ **:** M_{ID} ={3,4}

Step $_{12}$ **:** y=2

Step $_{13.2}$ **:** SS_{22} ={2,3}

Step $_{13.4.2}$ **:** b=2

Step $_{13.5}$ **:** I_{ID} ={1,2}

Step $_{10}$ **:** x=7

Step $_{11.2}$ **:** SS_{11}={7,8,9}

Step $_{11.3}$ **:** a=9

Step $_{11.5}$ **:** M_{ID} ={3,4,9}

Step $_{12}$ **:** y=4

Step $_{13.2}$ **:** SS_{22} ={4,5,6}

Step $_{13.3}$ **:** b=6

Step $_{13.5}$ **:** I_{ID} ={1,2,6}

Step $_{12}$ **:** y=8

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Step$_{13.2}$: SS_{22} ={8,9,10}

Step$_{13.3}$: b=10

Step$_{13.5}$: I_{ID} ={1,2,6,10}

Step$_{14}$: End

OUTPUT: I_{ID} ={1,2,6,10} is the required inverse independent dominating set w.r.t a minimum independent dominating set M_{ID} ={3,4,9} .

5. Main Results

Theorem 5.1: *Let* $i \in M_{ID}$, *If*

a) The interval $'i'$ *have more than one intersecting intervals which is less than i in* I_{ID} .

b) There is no $'j'$ *which is left intersecting interval to* i *or contained in* i *and* k *which is right intersecting interval to* i *or contains* i *such that* j *and* k *intersects.*

$$\text{Then } | M_{ID} |<| I_{ID} |.$$

Proof. Given that $i \in M_{ID}$ such that the interval $'i'$ have more than one intersecting interval in I_{ID} .

Suppose, the interval $'i'$ have two intersecting intervals a & $b < i$ in I_{ID} and also, b is the only intersecting interval to $j \in M_{ID}$ which is not possible because of *(b)*.

Therefore, interval $'i'$ is having two intersecting intervals $a, b \in I_{ID}$

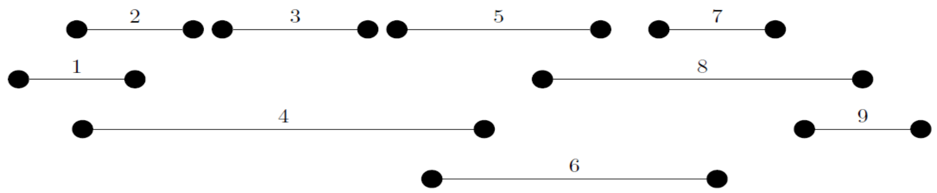
$\therefore$ Cardinality of i is less than cardinality of $\{a,b\}$

i.e., $1 < 2$

Similarly, we can prove the result if $'i'$ have more than two intersecting intervals

$$\therefore |M_{ID}|<|I_{ID}|.$$

Example:



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Theorem 5.2: *Let $i \in M_{ID}$, If the interval $'i'$ have two intersecting intervals in I_{ID} such that atleast one of these interval which is greater than $'i'$* then $|M_{ID}|=|I_{ID}|$.

Proof. Given that $i \in M_{ID}$ and the interval $'i'$ have two intersecting intervals å a or $b > i$ in I_{ID}.

Suppose $b \in I_{ID}$ is the only intersecting interval to $j \in M_{ID}$, then the cardinality of $\{a,b\}$ is equal to the cardinality of $\{i, j\}$.

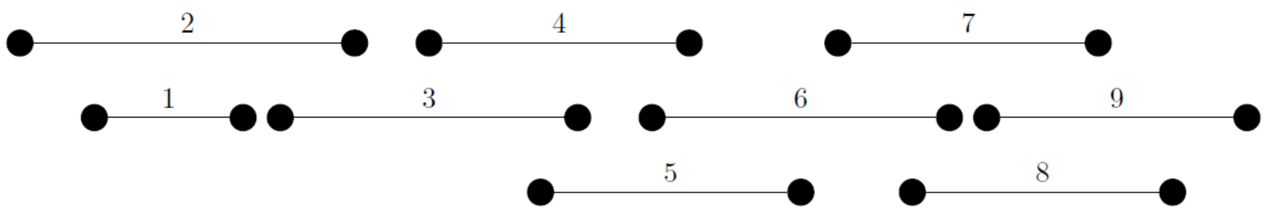
$$i.e., |\{a,b\}|=|\{i, j\}|$$

$$2=2.$$

Even though, $i \in M_{ID}$ have two intersecting intervals $a, b \in I_{ID}$ there is no increment in the cardinality of I_{ID}.

$$\therefore\ |M_{ID}|=|I_{ID}|$$

Example:



Theorem 5.3: *If i_p intersecting i_{p+1} for all $p=0,1,2,.....n-2$ where i_p and i_{p+1} are the intervals and $|M_{ID}|<|I_{ID}|$ then, for each $b \in I_{ID}$ there exists at least one $a \in M_{ID}$ such that $'b'$ is strongly dominated by $'a'$.*

Proof. Given that, i_p intersecting i_{p+1} for all $p=0,1,2,....,n-2$ where i_p and i_{p+1} are intervals then there is atleast one interval i_k to the left of $i_p\ (\neq 1)$ that intersects i_p and atleast one interval $i_l \neq i_k$ to the right of i_p that intersects i_p. Also, $|M_{ID}|<|I_{ID}|$.

Suppose, for each $b \in I_{ID}$ there exists atleast one $a \in M_{ID}$ such that $'b'$ is strictly weak dominated by $'a'$.

Let $M_{ID}=\{a_1, a_2, ..., a_m\}$ and $I_{ID}=\{b_1, b_2, ...b_n\}$ such that $m < n$. Then, each of b_j is strictly weak dominated by atleast one a_i, where $i=1,2,......,m$ and $j=1,2,.....,n$.

$$\Rightarrow deg(a_i) > deg(b_j)$$

$$\Rightarrow |\cup_{i=1}^{m} a_i| > |\cup_{j=1}^{n} b_j|$$

$$\Rightarrow |M_{ID}| > |I_{ID}|$$

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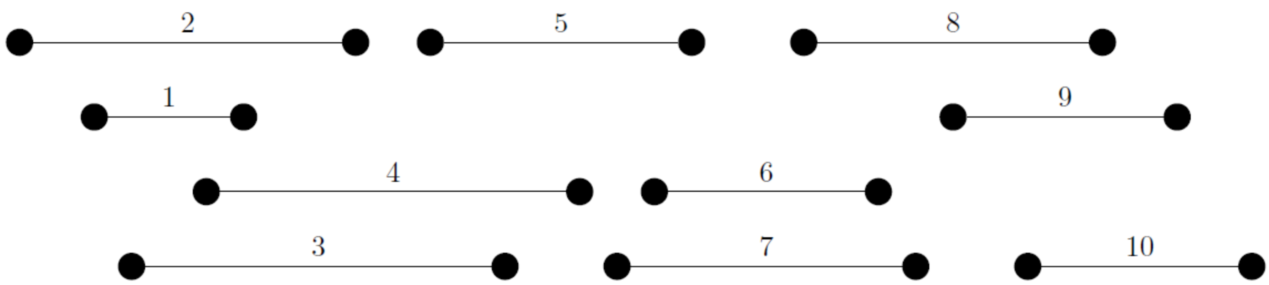
This contradicts the definition of inverse independent domination.

Hence, our assumption is wrong that, for each $b \in I_{ID}$ there exists atleast one $a \in M_{ID}$ such that $'b'$ is strictly weak dominated by $'a'$.

Therefore, $'b'$ is strongly dominated by $'a'$.

Example:



Theorem 5.4: *In G, let I_D be the minimum independent dominating set. Then $i(G) = i'(G)$ if $\forall\ b \in I_D$, $< N[b] >$ is a complete graph of order minimum two.*

Proof. Let $I_D = \{a_1, a_2, ... a_n\}$ be a minimum independent dominating set and $b_1, b_2, ... b_n$ are the vertices adjacent to $a_1, a_2, ... a_n$ respectively. By hypothesis, for every vertex $a_i \in I_D$ the graph $< N[a_i] >$ is complete, where $i = 1, 2, ... n$.

Then,

$$N[a_i] \subset N[b_i].$$

Therefore,

$$V(G) = N[a_1] \cup N[a_2] \cup .... \cup N[a_n] \subset N[b_1] \cup N[b_2] \cup .... \cup N[b_n] = V(G)$$

Hence, $\{b_1, b_2, ... b_n\}$ is an inverse independent dominating set.

Therefore,

$$i'(G) = |I_D| = i(G).$$

Hence, the proof.

Next, we discuss about the family of independent dominating set in Theorem 5.5.

Theorem 5.5: *In G, ζ denotes the family of minimum independent dominating sets. If $\forall\ I_D \in \zeta$, $V - I_D$ is an independent dominating, then $i'(G) + i(G) = p$.*

Proof. In G, since $V-I_D$ is an independent dominating for every minimum independent dominating set I_D, therefore $V-I_D$ itself is a minimum inverse independent dominating set. Therefore,

$$i(G)+i'(G) = p.$$

Theorem 5.6: *If a complete graph G of order* $p \geq 2$, *then* $i'(G) = \alpha(G) = 1$.

Proof. Since the graph G is a complete graph, the vertices in it are adjacent among themselves. Therefore any singleton set can be a minimum independent dominating set. Assume that, a vertex $'v'$ is a minimum independent dominating set in I_D of G, and then any other vertex $'u'$ is a minimum independent dominating set in $V-I_D$. At that point, $|I_D'|$ is one. That is $i'(G) = 1$. Also, for any complete graph, the number of elements in the largest independent set, $\alpha(G)=1$. Thus, $i'(G) = \alpha(G) = 1$.

Theorem 5.7: *If each non-end vertex of a graph G is adjacent to at least one end vertex, then* $i'(G) = \alpha(G)$.

Proof. Given, all the non-end vertices are adjacent to at least one end vertex. Then the graph G will belongs to one of the following:

(i) A graph in which each non-end vertices is adjacent to precisely one end vertex.

(ii) A graph in which some will be adjacent to precisely one end vertex and some will be adjacent to more than one end vertex.

(iii) A graph in which each non-end vertex is adjacent to more than one end vertex to dominate these end vertices we need to choose either support vertices or end vertices. In all these cases, the number of end vertices constitutes the inverse independent dominating set and also the maximum independent set. Thus, $i'(G) = \alpha(G)$.

Theorem 5.8: *Let G be a (p,q) graph with* $i(G) = i'(G)$, *then* $2p-3\ i(G) \leq q$.

Proof. Let I_D and I_D' be minimum independent dominating and inverse independent dominating sets of G respectively,then

$$q \geq 2\,|V(G) - I_D - I_D'| + |I_D|$$

$$= 2(p-2i(G)) + i(G)$$

$$= 2p - 3i(G).$$

Corollary 5.9: *If G be a (p,q) graph with no isolated vertices, then* $i'(G) \geq (2p-q)/3$.

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