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Y-index of four join operations of graphs

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For a molecular graph G, the Y-index is defined as the sum of fourth powers of degrees of all vertices of the graph. The Y-index is defined as $\mathrm{Y(G)} = \sum_{\mathrm{u \in V(G)}} \mathrm{d_G(u)^4} = \sum_{\mathrm{uv \in E(G)}} [\mathrm{d_G(u)^3 + d_G(v)^3}]$ and the Y −coindex is defined as $\overline{\mathrm{Y}}(\mathrm{G}) = \sum_{\mathrm{uv \notin E(G)}} [\mathrm{d_G(u)^3 + d_G(v)^3}]$. In this paper, we obtain the explicit expressions for Y −index and Y −coindex of four join operations of graphs such as vertex and edge S-join, vertex and edge R-join, vertex and edge Q-join and vertex and edge T-join of two graphs.

Keywords: Zagreb index; F-index; Y-index ; Join operations.

Mathematics Subject Classification 2020: 05C07,05C76,92E10

1. Introduction

All graphs considered here are simple, connected, finite and undirected. Let $G = (V(G), E(G))$ be a connected graph of order n with $|E(G)| = m$ edges. The degree of a vertex $u \in V(G)$, denoted by $d_G(u)$, is the number of egdes incident to u. The *join* of two graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ is the graph, denoted by $G_1 + G_2$, with the vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv | u \in V(G_1), v \in V(G_2)\}$.

In chemical graph theory, different types of topological indices have different types of applications in isomer discrimination, QSAR/QSPR investigation, pharmaceutical drug design and many more. Amid various degree-based topological indices, one of the most studied topological index, introduced by Gutman and Trinajstic', is zagreb index [7]. The Zagreb indices are defined as $M_1(G) = \sum_{v \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$ and $M_2(G) = \sum_{u,v \in V(G)} d_G(u) d_G(v)$.

In [5], Furtula and Gutman investigated forgotten topological index or the F −*index* of a graph G which is defined as $F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]$. The *Y-index* was introduced by Abdu Alameri et.al. in [1] which is defined as

$$Y(G) = \sum_{u \in V(G)} d_G(u)^4 = \sum_{uv \in E(G)} [d_G(u)^3 + d_G(v)^3].$$

In [2], Alemeri et al studied the Y −coindex of graph operations and its applications of molecular descriptors. The Y −coindex is defined as

$$\overline{Y}(G) = \sum_{uv \notin E(G)} [d_G(u)^3 + d_G(v)^3].$$

In this paper they showed that the predictive ability of this index is similar to that of first Zagreb index. There are various recent mathematical snd chemical studies of Y −index; see[2,13]. Recently V.S. Agnes et.al. studied about the Y −index and coindex of some derived graphs [12].

The degree distance was introduced by Dobrynin and Kochetova [4] and Gutman [8] as a weighted version of the Wiener index. The degree distance of graph G is defined as

$DD(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u,v)[d_G(u) + d_G(v)].$

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Some redefined versions of the Zagreb indices are given as follows:

$$ReZG_3(G) = \sum_{uv \in E(G)} d_G(u)d_G(v)[d_G(u) + d_G(v)],$$

$$ReZG_4(G) = \sum_{uv \in E(G)} d_G(u)d_G(v)[d_G(u)^2 + d_G(v)^2]$$

and

$$ReZG_4^2(G) = \sum_{uv \in E(G)} d_G(u)^2 d_G(v)^2$$

Throughout this paper, we use another index, denoted by $\xi_5(G)$, which is defined as

$$\xi_5(G) = \sum_{v \in V(G)} d_G(v)^5 = \sum_{uv \in E(G)} [d_G(u)^4 + d_G(v)^4].$$

In this paper, we obtain the explicit expressions of $Y-$index and coindex of four join operations of graphs such as vertex and edge S-join, vertex and edge R-join, vertex and edge Q-join and vertex and edge T-join of two graphs.

2. Preliminaries

• The *subdivision graph* $S(G)$ of a graph G is a graph obtained by inserting a new vertex onto each edge of G.

• *Vertex-semitotal graph* $R(G)$ of a graph G is a graph with vertex set $V(G) \cup E(G)$ and edge set $E(S) \cup E(L)$ which is obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it.

• E*dge-semitotal gaph* $Q(G)$ of a graph G is a graph with vertex set $V(G) \cup E(G)$ and $E(S) \cup E(L)$ which is obtained from G by inserting a new vertex into each edge of G and by joining with edges those pair of these new vertices which lie on adjacent edges of G.

• *Total graph* $T(G)$ of a graph G is a graph with vertex set $V(G) \cup E(G)$ and edge set $E(S) \cup E(G) \cup E(L)$ and any two vertices of $T(G)$ are adjacent if and only if they are either incident or adjacent in G.

3. Main Results

In this section we consider the $Y-$index of vertex and edge $\mathcal{F}s-$join of graphs related to different subdivision graphs such as $S, R, Q,$ and T respectively. Here $I(G)$ denotes the set of new vertices added while subdividing the graph G.

3.1 Vertex and edge S-Join of Graphs

In this subsection, first we start with vertex and edge $S-$join of graphs.

Vertex S-join of graph

Definition 3.1.1 1 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The vertex S-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \dot{\vee}_S G_2$, is obtained from $S(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of G_2.*

Example : The graph of vertex $S-$join of P_3 and P_3 are shown in Fig.1.

The degree of a vertex of $S-$join graph is given in the following lemma.

Lemma 3.1.1 2 [10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $w \in V(G_1 \dot{\vee} G_2)$ is*

$$d_{G_1 \dot{\vee} G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & \text{if } v \in V(G_1) \\ d_{G_2}(v) + n_1, & \text{if } v \in V(G_2) \\ 2, & \text{if } v \in I(G_1). \end{cases}$$

Theorem 3.1.1 3 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \dot{\vee}_S G_2) \quad = \quad & Y(G_1) + Y(G_2) + 4n_2F(G_1) + 4n_1F(G_2) + 6n_2^2M_1(G_1) \\ & +6n_1^2M_1(G_2) + 8m_1n_2^3 + 8m_2n_1^3 + n_1n_2^4 + n_2n_1^4 + 16m_1. \end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1 \dot{\vee}_S G_2) \quad &= \quad \sum_{v \in V(G_1 \dot{\vee}_S G_2)} d_{G_1 \dot{\vee}_S G_2}(v)^4 \\ &= \quad \sum_{v \in V(G_1)} d_{G_1 \dot{\vee}_S G_2}(v)^4 + \sum_{v \in V(G_2)} d_{G_1 \dot{\vee}_S G_2}(v)^4 + \sum_{v \in I(G_1)} d_{G_1 \dot{\vee}_S G_2}(v)^4 \\ &= \quad \sum_{v \in V(G_1)} (d_{G_1}(v) + n_2)^4 + \sum_{v \in V(G_2)} (d_{G_2}(v) + n_1)^4 + \sum_{v \in I(G_1)} (2)^4 \\ &= \quad \sum_{v \in V(G_1)} (d_{G_1}(v)^4 + 4d_{G_1}(v)^3n_2 + 6d_{G_1}(v)^2n_2^2 + 4d_{G_1}n_2^3 + n_2^4) \\ & \quad + \sum_{v \in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3n_1 + 6d_{G_2}(v)^2n_1^2 + 4d_{G_2}n_1^3 + n_1^4) \\ & \quad + \sum_{v \in I(G_1)} 16 \\ &= \quad Y(G_1) + Y(G_2) + 4n_2F(G_1) + 4n_1F(G_2) + 6n_2^2M_1(G_1) \\ & \quad +6n_1^2M_1(G_2) + 8m_1n_2^3 + 8m_2n_1^3 + n_1n_2^4 + n_2n_1^4 + 16m_1 \end{aligned}$$

which is the desired result.

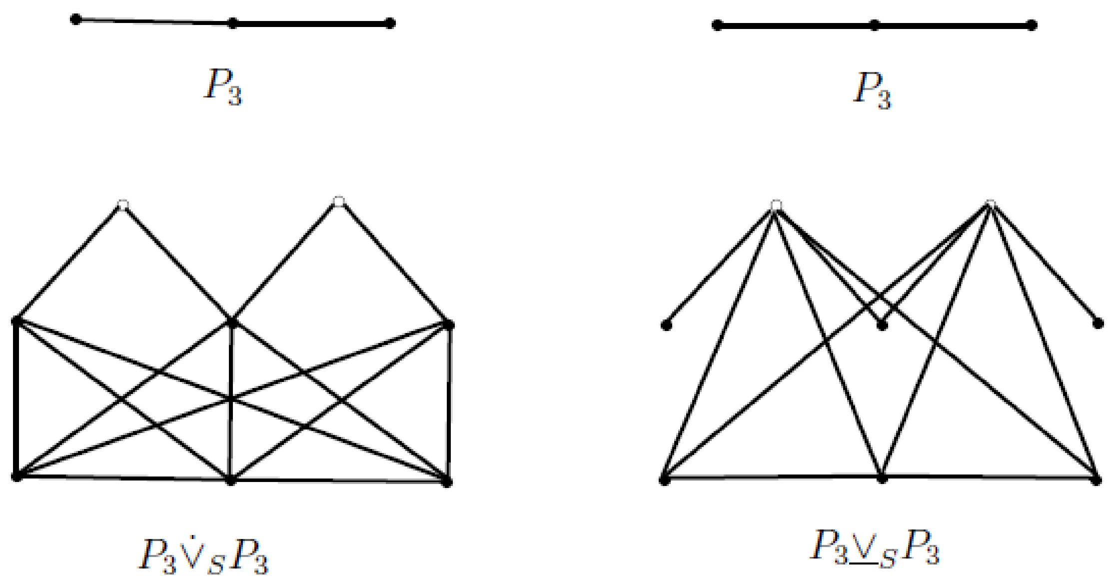

Fig. 1 The vertex S-join and edge S-join of graphs

Edge S-join of graph

Definition 3.1.2 4 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The edge S-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \underline{\vee}_S G_2$, is obtained from $S(G_1)$ and G_2 by joining each vertex of $I(G_1)$ with every vertex of G_2.*

Example : The graph of edge S −join of P_3 and P_3 is shown in Fig.1.

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The degree of a vertex of edge S −join graph is given in the following lemma.

Lemma 3.1.2 [10] **5** *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \underline{\vee}_S G_2)$ is*

$$d_{G_1\underline{\vee}_S G_2}(v) = \begin{cases} d_{G_1}(v), & \text{if } v \in V(G_1) \\ d_{G_2}(v) + m_1, & \text{if } v \in V(G_2) \\ 2 + n_2, & \text{if } v \in I(G_1). \end{cases}$$

Theorem 3.1.2 6 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \underline{\vee}_S G_2) \quad = \quad & Y(G_1) + Y(G_2) + m_1(2+n_2)^4 + 4m_1F(G_2) + 6m_1^2M_1(G_2) \\ & +8m_2m_1^3 + n_2m_1^4. \end{aligned}$$

Proof. From definition of Y −index, we have

$$\begin{aligned} Y(G_1 \underline{\vee}_S G_2) \quad &= \quad \textstyle\sum_{v\in V(G_1\underline{\vee}_S G_2)} d_{G_1\underline{\vee}_S G_2}(v)^4 \\ &= \quad \textstyle\sum_{v\in V(G_1)} d_{G_1\underline{\vee}_S G_2}(v)^4 + \sum_{v\in V(G_2)} d_{G_1\underline{\vee}_S G_2}(v)^4 + \sum_{v\in I(G_1)} d_{G_1\underline{\vee}_S G_2}(v)^4 \\ &= \quad \textstyle\sum_{v\in V(G_1)} d_{G_1}(v)^4 + \sum_{v\in V(G_2)} (d_{G_2}(v)+m_1)^4 + \sum_{v\in I(G_1)} (2+n_2)^4 \\ &= \quad \textstyle\sum_{v\in V(G_1)} d_{G_1}(v)^4 + \sum_{v\in I(G_1)} (2+n_2)^4 + \sum_{v\in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3m_1 \\ &\quad +6d_{G_2}(v)^2m_1^2 + 4d_{G_2}m_1^3 + m_1^4) \\ &= \quad Y(G_1) + m_1(2+n_2)^4 + Y(G_2) + 4m_1F(G_2) + 6m_1^2M_1(G_2) \\ &\quad +8m_2m_1^3 + n_2m_1^4 \end{aligned}$$

which is the desired result.

3.2 Vertex and edge R-Join of Graphs

In this subsection, we obtain vertex and edge R −join of graphs.

Vertex R-join of graph

Definition 3.2.1 7 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The vertex R-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \dot{\vee}_R G_2$, is obtained from $R(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of G_2.*

Example: The graph of vertex R −join of P_3 and P_3 is given in Fig.2.

The degree of a vertex of R −join graph is given in the following lemma.

Lemma 3.2.1 8 [10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \dot{\vee}_R G_2)$ is*

$$d_{G_1\dot{\vee}_R G_2}(v) = \begin{cases} 2d_{G_1}(v) + n_2, & \text{if } v \in V(G_1) \\ d_{G_2}(v) + n_1, & \text{if } v \in V(G_2) \\ 2, & \text{if } v \in I(G_1). \end{cases}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
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Theorem 3.2.1 9 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \dot{\vee}_R G_2) \quad = \quad & 16Y(G_1) + Y(G_2) + 32n_2F(G_1) + 4n_1F(G_2) + 24n_2^2M_1(G_1) \\ & +6n_1^2M_1(G_2) + 16n_2^3m_1 + 8n_1^3m_2 + n_2^4n_1 + n_1^4n_2 + 16m_1. \end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1 \dot{\vee}_R G_2) \quad = \quad & \textstyle\sum_{v\in V(G_1\dot{\vee}_R G_2)} d_{G_1\dot{\vee}_R G_2}(v)^4 \\ = \quad & \textstyle\sum_{v\in V(G_1)} d_{G_1\dot{\vee}_R G_2}(v)^4 + \sum_{v\in V(G_2)} d_{G_1\dot{\vee}_R G_2}(v)^4 + \sum_{v\in I(G_1)} d_{G_1\dot{\vee}_R G_2}(v)^4 \\ = \quad & \textstyle\sum_{v\in V(G_1)} (2d_{G_1}(v) + n_2)^4 + \sum_{v\in V(G_2)} (d_{G_2}(V) + n_1)^4 + \sum_{v\in I(G_1)} (2)^4 \\ = \quad & \textstyle\sum_{v\in V(G_1)} (16d_{G_1}(v)^4 + 32d_{G_1}(v)^3n_2 + 24d_{G_1}(v)^2n_2^2 + 8d_{G_1}(v)n_2^3 + n_2^4) \\ & \textstyle+\sum_{v\in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3n_1 + 6d_{G_2}(v)^2n_1^2 + 4d_{G_2}(v)n_1^3 + n_1^4) \\ & \textstyle+\sum_{v\in I(G_1)} 16 \\ = \quad & 16Y(G_1) + 32n_2F(G_1) + 24n_2^2M_1(G_1) + 16n_2^3m_1 + n_2^4n_1 + Y(G_2) \\ & +4n_1F(G_2) + 6n_1^2M_1(G_2) + 8n_1^3m_2 + n_1^4n_2 + 16m_1 \end{aligned}$$

which is desired result.

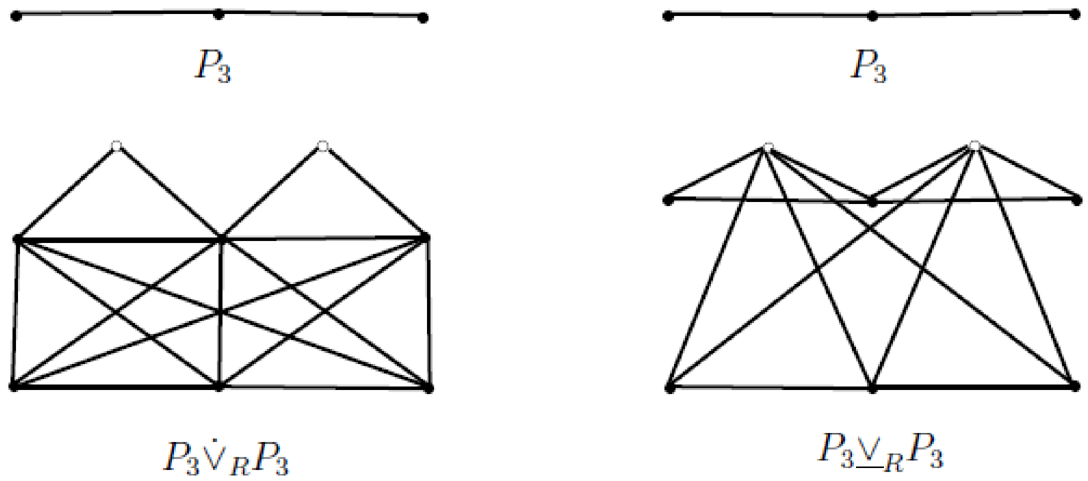

Fig. 2 The vertex R-join and edge R-join of graphs

Edge R-join of graph

Definition 3.2.2 10 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The edge R-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \underline{\vee}_R G_2$, is obtained from $R(G_1)$ and G_2 by joining each vertex of $I(G_1)$ with every vertex of G_2.*

Example : The graph of edge R −join of P_3 and P_3 in given in Fig.2.

The degree of a vertex of edge R −join graph is given in the following lemma.

Lemma 3.2.2 11 [10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \underline{\vee}_R G_2)$ is*

JOURNAL OF ALGEBRAIC STATISTICS
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$$d_{G_1 \underline{\vee}_R G_2}(v) = \begin{cases} 2d_{G_1}(v), & \text{if } v \in V(G_1) \\ d_{G_2}(v) + m_1, & \text{if } v \in V(G_2) \\ 2 + n_2, & \text{if } v \in I(G_1). \end{cases}$$

Theorem 3.2.2 12 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \underline{\vee}_R G_2) &= 16Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 \\ &\quad + n_2m_1^4 + m_1(n_2+2)^4. \end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1 \underline{\vee}_R G_2) &= \textstyle\sum_{v \in V(G_1 \underline{\vee}_R G_2)} d_{G_1 \underline{\vee}_R G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} d_{G_1 \underline{\vee}_R G_2}(v)^4 + \sum_{v \in V(G_2)} d_{G_1 \underline{\vee}_R G_2}(v)^4 + \sum_{v \in I(G_1)} d_{G_1 \underline{\vee}_R G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} (2d_{G_1}(v))^4 + \sum_{v \in V(G_2)} (d_{G_2}(v) + m_1)^4 + \sum_{v \in I(G_1)} (n_2+2)^4 \\ &= \textstyle\sum_{v \in V(G_1)} 16d_{G_1}(v)^4 + \sum_{v \in I(G_2)} (n_2+2)^4 + \sum_{v \in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3m_1 \\ &\quad + 6d_{G_2}(v)^2m_1^2 + 4d_{G_2}(v)m_1^3 + m_1^4) \\ &= 16Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 \\ &\quad + n_2m_1^4 + m_1(n_2+2)^4 \end{aligned}$$

which is the desired result.

3.3 Vertex and edge Q-Join of Graphs

In this subsection, we obtain vertex and edge Q −join of graphs.

Vertex Q-join of graph

Definition 3.3.1 13 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The vertex Q-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \dot{\vee}_Q G_2$, is obtained from $Q(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of G_2.*

Example : The graph of vertex Q −join of P_3 and P_3 in Fig.3.

The degree of a vertex of Q −join graph is in the following lemma.

Lemma 3.3.1 14[10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \dot{\vee}_Q G_2)$ is*

$$d_{G_1 \dot{\vee}_Q G_2}(v) = \begin{cases} d_{G_1}(v) + n_2, & \text{if } v \in V(G_1) \\ d_{G_2}(v) + n_1, & \text{if } v \in V(G_2) \\ d_{G_1}(u) + d_{G_2}(v), & \text{if } v \in I(G_1). \end{cases}$$

Theorem 3.3.1 15 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \dot{\vee}_Q G_2) &= Y(G_1) + 4F(G_1)n_2 + 6M_1(G_1)n_2^2 + 8m_1n_2^3 + n_1n_2^4 + Y(G_2) \\ &\quad + 4F(G_2)n_1 + 6M_1(G_2)n_1^2 + 8m_2n_1^3 + n_2n_1^4 + \xi_5(G) \\ &\quad + 4ReZ_4(G_1) + 6ReZ_4^2(G_1) \end{aligned}$$

Proof. From the definition of Y −index, we have

$$
\begin{aligned}
Y(G_1 \dot{\vee}_Q G_2) &= \sum_{v\in V(G_1\dot{\vee}_Q G_2)} d_{G_1\dot{\vee}_Q G_2}(v)^4 \\
&= \sum_{v\in V(G_1)} d_{G_1\dot{\vee}_Q G_2}(v)^4 + \sum_{v\in V(G_2)} d_{G_1\dot{\vee}_Q G_2}(v)^4 + \sum_{v\in I(G_1)} d_{G_1\dot{\vee}_Q G_2}(v)^4 \\
&= \sum_{v\in V(G_1)} (d_{G_1}(v)+n_2)^4 + \sum_{v\in V(G_2)} (d_{G_2}(v)+n_1)^4 + \sum_{v\in I(G_1)} d_{I(G_1)}(V)^4 \\
&= \sum_{v\in V(G_1)} d_{G_1}(v)^4 + 4d_{G_1}(v)^3 n_2 + 6d_{G_1}(v)^2 n_2^2 + 4d_{G_1}(v) n_2^3 + n_2^4 \\
&\quad + \sum_{v\in V(G_2)} d_{G_2}(v)^4 + 4d_{G_2}(v)^3 n_1 + 6d_{G_2}(v)^2 n_1^2 + 4d_{G_2}(v) n_1^3 + n_1^4 \\
&\quad + \sum_{uv\in E(G_1)} (d_{G_1}(u) + d_{G_1}(v))^4
\end{aligned}
$$

$$
\begin{aligned}
&= Y(G_1) + 4F(G_1)n_2 + 6M_1(G_1)n_2^2 + 8m_1 n_2^3 + n_1 n_2^4 + Y(G_2) \\
&\quad +4F(G_2)n_1 + 6M_1(G_2)n_1^2 + 8m_2 n_1^3 + n_2 n_1^4 \\
&\quad + \sum_{uv\in E(G_1)} (d_{G_1}(u)^4 + 4d_{G_1}(u)^3 d_{G_1}(v) \\
&\quad +6d_{G_1}(u)^2 d_{G_1}(v)^2 + 4d_{G_1}(u) d_{G_1}(v)^3 + d_{G_1}(v)^4) \\
&= Y(G_1) + 4F(G_1)n_2 + 6M_1(G_1)n_2^2 + 8m_1 n_2^3 + n_1 n_2^4 + Y(G_2) \\
&\quad +4F(G_2)n_1 + 6M_1(G_2)n_1^2 + 8m_2 n_1^3 + n_2 n_1^4 + \xi_5(G) \\
&\quad +4ReZ_4(G_1) + 6ReZ_4^2(G_1)
\end{aligned}
$$

which is the desired result.

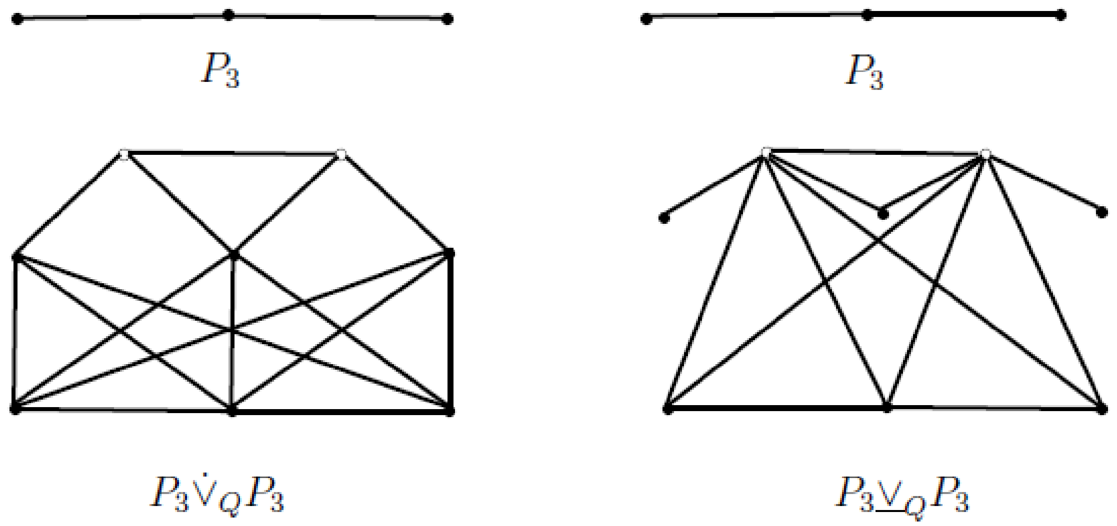

Fig 3. The vertex Q-join and edge Q-join of graphs

Edge Q-join of graph

Definition 3.3.2 16 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The edge Q −join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \underline{\vee}_Q G_2$, is obtained from $Q(G_1)$ and G_2 by joining each vertex of $I(G_1)$ with every vertex of G_2.*

Example : The graph of edge Q −join of P_3 and P_3 in Fig.3.

The degree of a vertex of edge R −join graph is given in the following lemma.

Lemma 3.3.2 17 [10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \underline{\vee}_Q G_2)$ is*

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
https://publishoa.com
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$$d_{G_1 \underline{\vee}_Q G_2}(v) = \begin{cases} d_{G_1}(v), & \text{if } v \in V(G_1) \\ d_{G_2}(v) + m_1, & \text{if } v \in V(G_2) \\ d_{G_1}(u) + d_{G_1}(v) + n_2, & \text{if } v \in I(G_1). \end{cases}$$

Theorem 3.3.2 18 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \underline{\vee}_Q G_2) &= Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \xi_5(G_1) + ReZ_4(G_1) + ReZ_4^2(G_1) + Y(G_1)n_2 + 12DD(G_1)n_2 \\ &\quad + 6F(G_1)n_2^2 + 12M_2(G_1)n_2^2 + 4M_1(G_1)n_2^3 + n_1n_2^4. \end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1 \underline{\vee}_Q G_2) &= \textstyle\sum_{v \in V(G_1 \underline{\vee}_Q G_2)} d_{G_1 \underline{\vee}_Q G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} d_{G_1 \underline{\vee}_Q G_2}(v)^4 + \sum_{v \in V(G_2)} d_{G_1 \underline{\vee}_Q G_2}(v)^4 + \sum_{v \in I(G_1)} d_{G_1 \underline{\vee}_Q G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} (d_{G_1}(v))^4 + \sum_{v \in V(G_2)} (d_{G_2}(v) + m_1)^4 + \sum_{uv \in E(G_1)} (d_{G_1}(u) + d_{G_1}(v) + n_2)^4 \\ &= \textstyle\sum_{v \in V(G_1)} d_{G_1}(v)^4 + \sum_{v \in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3m_1 + 6d_{G_2}(v)^2m_1^2 \\ &\quad + 4d_{G_2}(v)m_1^3 + m_1^4) + \textstyle\sum_{uv \in E(G_1)} ((d_{G_1}(u) + d_{G_1}(v))^4 + 4(d_{G_1}(u) + d_{G_1}(v))^3n_2 \\ &\quad + 6(d_{G_1}(u) + d_{G_1}(v))^2n_2^2 + 4(d_{G_1}(u) + d_{G_2}(v))n_2^3 + n_2^4) \\ &= Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u)^4 + d_{G_1}(v)^4) + 4\sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v)(d_{G_1}(u)^2 + d_{G_1}(v)^2) \\ &\quad + 6\textstyle\sum_{uv \in E(G_1)} d_{G_1}(u)^2d_{G_1}(v)^2 + 4n_2\sum_{uv \in E(G_1)} (d_{G_1}(u)^3 + d_{G_1}(v)^3) \\ &\quad + 12n_2\textstyle\sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v)(d_{G_1}(u) + d_{G_1}(v)) \\ &\quad + 6n_2^2\textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u)^2 + d_{G_1}(v)^2) + 12n_2^2\sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v) \\ &\quad + 4n_2^3\textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u) + d_{G_1}(v)) + n_1n_2^4 \\ &= Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \xi_5(G_1) + ReZ_4(G_1) + ReZ_4^2(G_1) + Y(G_1)n_2 + 12DD(G_1)n_2 \\ &\quad + 6F(G_1)n_2^2 + 12M_2(G_1)n_2^2 + 4M_1(G_1)n_2^3 + n_1n_2^4 \end{aligned}$$

which is desired result.

3.4 Vertex and edge T-Join of Graphs

In this subsection, we obtain vertex and edge T −join of graphs.

Vertex T-join of graph

Definition 3.4.1 19 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The vertex T-join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \dot{\vee}_T G_2$, is obtained from $T(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of G_2.*

Example : The graph of vertex T −join of P_3 and P_3 in Fig.4.

The degree of a vertex of T −join graph is given in the following lemma.

Lemma 3.4.1 20[10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \dot{\vee}_T G_2)$ is*

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
https://publishoa.com
ISSN: 1309-3452

$$d_{G_1\dot{\vee}_T G_2}(v)=\begin{cases} 2d_{G_1}(v)+n_2, & \text{if } v\in V(G_1)\\ d_{G_2}(v)+n_1, & \text{if } v\in V(G_2)\\ d_{G_1}(u)+d_{G_2}(v), & \text{if } v\in I(G_1).\end{cases}$$

Theorem 3..4.1 21 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i\in\{1,2\}$, then*

$$\begin{aligned} Y(G_1\dot{\vee}_T G_2) \quad = \quad & 16Y(G_1)+32F(G_1)n_2+24M_1(G_1)n_2^2+16m_1n_2^3+n_1n_2^4\\ & +Y(G_2)+4F(G_2)n_1+6M_1(G_2)n_1^2+8m_2n_1^3+n_2n_1^4\\ & +\xi_5(G_1)+4ReZ_4(G_1)+6ReZ_4^2(G_1).\end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1\dot{\vee}_T G_2) \quad = \quad & \textstyle\sum_{v\in V(G_1\dot{\vee}_T G_2)} d_{G_1\dot{\vee}_T G_2}(v)^4\\ = \quad & \textstyle\sum_{v\in V(G_1)} d_{G_1\dot{\vee}_T G_2}(v)^4+\sum_{v\in V(G_2)} d_{G_1\dot{\vee}_T G_2}(v)^4+\sum_{v\in I(G_1)} d_{G_1\dot{\vee}_T G_2}(v)^4\\ = \quad & \textstyle\sum_{v\in V(G_1)} (2d_{G_1}(v)+n_2)^4+\sum_{v\in V(G_2)} (d_{G_2}+n_1)^4+\sum_{v\in I(G_1)} d_{I(G_1)}(v)^4\\ = \quad & \textstyle\sum_{v\in V(G_1)} (16d_{G_1}(v)^4+32d_{G_1}(v)^3n_2+24d_{G_1}(v)^2n_2^2+8d_{G_1}(v)n_2^3+n_2^4)\\ & \textstyle+\sum_{v\in V(G_2)} (d_{G_2}(v)^4+4d_{G_2}(v)^3n_1+6d_{G_2}(v)^2n_1^2+4d_{G_2}(v)n_1^3+n_1^4)\\ & \textstyle+\sum_{uv\in E(G_1)} (d_{G_1}(u)+d_{G_1}(v))^4\\ = \quad & \textstyle\sum_{v\in V(G_1)} (16d_{G_1}(v)^4+32d_{G_1}(v)^3n_2+24d_{G_1}(v)^2n_2^2+8d_{G_1}(v)n_2^3+n_2^4)\\ & \textstyle+\sum_{v\in V(G_2)} (d_{G_2}(v)^4+4d_{G_2}(v)^3n_1+6d_{G_2}(v)^2n_1^2+4d_{G_2}(v)n_1^3+n_1^4)\\ & \textstyle+\sum_{uv\in E(G_1)} (d_{G_1}(u)^4+d_{G_1}(v)^4)+6\sum_{uv\in E(G_1)} d_{G_1}(u)^2d_{G_1}(v)^2\\ & \textstyle+4\sum_{uv\in E(G_1)} d_{G_1}(u)d_{G_1}(v)(d_{G_1}(u)^2+d_{G_1}(v)^2)\\ = \quad & 16Y(G_1)+32F(G_1)n_2+24M_1(G_1)n_2^2+16m_1n_2^3+n_1n_2^4\\ & +Y(G_2)+4F(G_2)n_1+6M_1(G_2)n_1^2+8m_2n_1^3+n_2n_1^4\\ & +\xi_5(G_1)+4ReZ_4(G_1)+6ReZ_4^2(G_1)\end{aligned}$$

which is desired result.

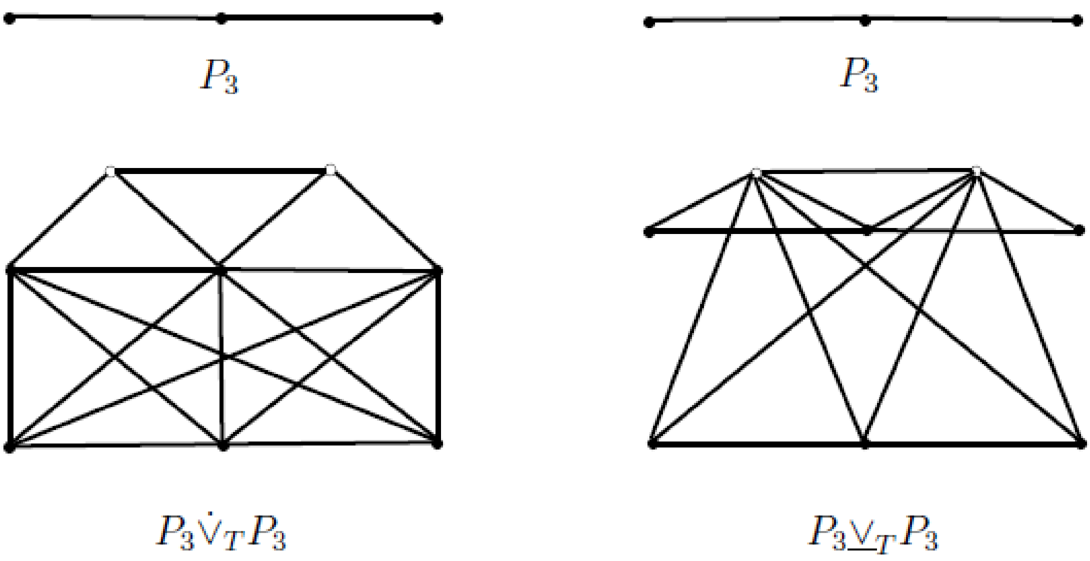

Fig 4. The vertex T-join and edge T-join of graphs

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
https://publishoa.com
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Edge T-join of graph

Definition 3.4.2 22 *Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The edge T −join of two vertex disjoint graphs G_1 and G_2, denoted by $G_1 \underline{\vee}_T G_2$, is obtained from $T(G_1)$ and G_2 by joining each vertex of $I(G_1)$ with every vertex of G_2.*

Example : The graph of edge T −join of P_3 and P_3 in Fig.4.

The degree of a vertex of edge T −join graph is given in the following lemma.

Lemma 3.4.2 23[10] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then the degree of $v \in V(G_1 \underline{\vee}_T G_2)$ is*

$$d_{G_1 \underline{\vee}_T G_2}(v) = \begin{cases} 2d_{G_1}(v), & \text{if } v \in V(G_1) \\ d_{G_2}(v) + m_1, & \text{if } v \in V(G_2) \\ d_{G_1}(u) + d_{G_1}(v) + n_2, & \text{if } e = (u,v), e \in I(G_1). \end{cases}$$

Theorem 3.4.2 24 *If G_1 and G_2 are two connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$, then*

$$\begin{aligned} Y(G_1 \underline{\vee}_T G_2) &= 16Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \xi_5(G_1) + ReZ_4(G_1) + ReZ_4^2(G_1) + Y(G_1)n_2 + 12DD(G_1)n_2 \\ &\quad + 6F(G_1)n_2^2 + 12M_2(G_1)n_2^2 + 4M_1(G_1)n_2^3 + n_1n_2^4. \end{aligned}$$

Proof. From the definition of Y −index, we have

$$\begin{aligned} Y(G_1 \underline{\vee}_T G_2) &= \textstyle\sum_{v \in V(G_1 \underline{\vee}_T G_2)} d_{G_1 \underline{\vee}_T G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} d_{G_1 \underline{\vee}_T G_2}(v)^4 + \sum_{v \in V(G_2)} d_{G_1 \underline{\vee}_T G_2}(v)^4 + \sum_{v \in I(G_1)} d_{G_1 \underline{\vee}_T G_2}(v)^4 \\ &= \textstyle\sum_{v \in V(G_1)} (2d_{G_1}(v))^4 + \sum_{v \in (G_2)} (d_{G_2} + m_1)^4 \\ &\quad + \textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u) + d_{G_1}(v) + n_2)^4 \\ &= 16 \textstyle\sum_{v \in V(G_1)} d_{G_1}(v)^4 + \sum_{v \in V(G_2)} (d_{G_2}(v)^4 + 4d_{G_2}(v)^3m_1 + 6d_{G_2}(v)^2m_1^2 \\ &\quad + 4d_{G_2}(v)m_1^3 + m_1^4) + \textstyle\sum_{uv \in E(G_1)} ((d_{G_1}(u) + d_{G_1}(v))^4 + 4(d_{G_1}(u) + d_{G_1}(v))^3n_2 \\ &\quad + 6(d_{G_1}(u) + d_{G_1}(v))^2n_2^2 + 4(d_{G_1}(u) + d_{G_2}(v))n_2^3 + n_2^4) \\ &= 16Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u)^4 + d_{G_1}(v)^4) + 4\sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v)(d_{G_1}(u)^2 + d_{G_1}(v)^2) \\ &\quad + 6\textstyle\sum_{uv \in E(G_1)} d_{G_1}(u)^2d_{G_1}(v)^2 + 4n_2\sum_{uv \in E(G_1)} (d_{G_1}(u)^3 + d_{G_1}(v)^3) \\ &\quad + 12n_2 \textstyle\sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v)(d_{G_1}(u) + d_{G_1}(v)) \\ &\quad + 6n_2^2 \textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u)^2 + d_{G_1}(v)^2) + 12n_2^2 \sum_{uv \in E(G_1)} d_{G_1}(u)d_{G_1}(v) \\ &\quad + 4n_2^3 \textstyle\sum_{uv \in E(G_1)} (d_{G_1}(u) + d_{G_1}(v)) + n_1n_2^4 \\ &= 16Y(G_1) + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4 \\ &\quad + \xi_5(G_1) + ReZ_4(G_1) + ReZ_4^2(G_1) + Y(G_1)n_2 + 12DD(G_1)n_2 \\ &\quad + 6F(G_1)n_2^2 + 12M_2(G_1)n_2^2 + 4M_1(G_1)n_2^3 + n_1n_2^4 \end{aligned}$$

which is the desired result.

Example 3.4.1 25 *If C_n and C_m are two cycles of order $n \geq 3$ and size $m \geq 3$, respectively. then*

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
https://publishoa.com
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(i) $Y(C_n \dot{\vee}_S C_m) = nm(m^3+n^3) + 8nm(m^2+n^2) + 24nm(m+n) + 64nm + 32n + 16m.$

(ii) $Y(C_n \dot{\vee}_R C_m) = nm(m^3+n^3) + 4nm(3m^2+2n^2) + 3nm(18m+8n) + 140nm + 97n + 16m.$

(iii) $Y\left(C_n \dot{\vee}_Q C_m\right) = nm(m^3+n^3) + 4nm(3m^2+2n^2) + 3nm(18m+8n) + 140nm + 97n + 16m.$

(iv) $Y(C_n \dot{\vee}_T C_m) = nm(m^3+n^3) + 8nm(2m^2+n^2) + 8nm(12m+3n) + 288nm + 512n + 16m.$

3.5 Y-coindex of $\mathcal{F}$ −join of graphs

In this section, we compute the Y-coindex of vertex and edge $\mathcal{F}$ −join of graphs for different values of $\mathcal{F}$ as S, R, Q, T respectively.

Lemma 3.5.1 [3] **26** *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then*

$$\begin{aligned}(i)|V(G_1+G_2)| &= n_1+n_2,\\ (ii)|E(G_1+G_2)| &= m_1+m_2+n_1n_2.\end{aligned}$$

Theorem 3.5.1 27 [11] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then*

$$\begin{aligned}(i)F(G_1 \dot{\vee}_S G_2) &= F(G_1)+F(G_2)+3n_2M_1(G_1)+3n_1M_1(G_2)+6m_1n_2^2\\ &\quad +6m_2n_1^2+n_1n_2(n_1^2+n_2^2)+8m_1.\\ (ii)F(G_1 \dot{\vee}_R G_2) &= 8F(G_1)+F(G_2)+12n_2M_1(G_1)+3n_1M_1(G_2)+12m_1n_2^2\\ &\quad +6m_2n_1^2+n_1n_2(n_1^2+n_2^2)+8m_1.\\ (iii)F(G_1 \dot{\vee}_Q G_2) &= F(G_1)+F(G_2)+3n_2M_1(G_1)+3n_1M_1(G_2)+Y(G_1)\\ &\quad +3ReZG_3(G_1)+6m_1n_2^2+6m_2n_1^2+n_1n_2(n_1^2+n_2^2).\\ (iv)F(G_1 \dot{\vee}_T G_2) &= 8F(G_1)+F(G_2)+12n_2M_1(G_1)+3n_1M_1(G_2)+Y(G_1)\\ &\quad +3ReZG_3(G_1)+12m_1n_2^2+6m_2n_1^2+n_1n_2(n_1^2+n_2^2).\end{aligned}$$

Theorem 3.5.2 28 [2] *Let G be a simple graph with n vertices and m edges. Then*

$$\overline{Y}(G) = (n-1)F(G) - Y(G).$$

Theorem 3.5.3 29 *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then*

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 273-285
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$$
\begin{aligned}
(i)Y(G_1 \dot{\vee}_S G_2) \;=\;& (n_1+n_2-1)F(G_1)+F(G_2)+3n_2M_1(G_1)+3n_1M_1(G_2)+6m_1n_2^2\\
&+6m_2n_1^2+n_1n_2(n_1^2+n_2^2)+8m_1-Y(G_1)+Y(G_2)+4n_2F(G_1)\\
&+4n_1F(G_2)+6n_2^2M_1(G_1)+6n_1^2M_1(G_2)+8m_1n_2^3\\
&+8m_2n_1^3+n_1n_2^4+n_2n_1^4+16m_1\\
(ii)Y(G_1 \dot{\vee}_R G_2) \;=\;& (n_1+n_2-1)8F(G_1)+F(G_2)+12n_2M_1(G_1)+3n_1M_1(G_2)\\
&+12m_1n_2^2+6m_2n_1^2+n_1n_2(n_1^2+n_2^2)+8m_1-16Y(G_1)\\
&+32n_2F(G_1)+24n_2^2M_1(G_1)+16n_2^3m_1+n_2^4n_1+Y(G_2)\\
&+4n_1F(G_2)+6n_1^2M_1(G_2)+8n_1^3m_2+n_1^4n_2+16m_1\\
(iii)Y(G_1 \dot{\vee}_Q G_2) \;=\;& (n_1+n_2-1)F(G_1)+F(G_2)+3n_2M_1(G_1)+3n_1M_1(G_2)+Y(G_1)\\
&+3ReZG_3(G_1)+6m_1n_2^2+6m_2n_1^2+n_1n_2(n_1^2+n_2^2)-Y(G_1)\\
&+4F(G_1)n_2+6M_1(G_1)n_2^2+8m_1n_2^3+n_1n_2^4+Y(G_2)\\
&+4F(G_2)n_1+6M_1(G_2)n_1^2+8m_2n_1^3+n_2n_1^4+\xi_5(G)\\
&+4ReZ_4(G_1)+6ReZ_4^2(G_1)\\
(iv)Y(G_1 \dot{\vee}_T G_2) \;=\;& (n_1+n_2-1)8F(G_1)+F(G_2)+12n_2M_1(G_1)+3n_1M_1(G_2)+Y(G_1)\\
&+32F(G_1)n_2+24M_1(G_1)n_2^2+16m_1n_2^3+n_1n_2^4\\
&+Y(G_2)+4F(G_2)n_1+6M_1(G_2)n_1^2+8m_2n_1^3+n_2n_1^4\\
&+\xi_5(G_1)+4ReZ_4(G_1)+6ReZ_4^2(G_1).
\end{aligned}
$$

Theorem 3.5.3 30 [11] *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then*

$$
\begin{aligned}
(i)F(G_1 \underline{\vee}_S G_2) \;=\;& F(G_1)+F(G_2)+3m_1M_1(G_2)+6m_1^2m_2+m_1(n_2+2)^3+n_2m_1^3.\\
(ii)F(G_1 \underline{\vee}_R G_2) \;=\;& 8F(G_1)+F(G_2)+3m_1M_1(G_2)+6m_1^2m_2+m_1(n_2+2)^3+n_2m_1^3.\\
(iii)F(G_1 \underline{\vee}_Q G_2) \;=\;& F(G_1)+F(G_2)+3n_2^2M_1(G_1)+3m_1M_1(G_2)+Y(G_1)\\
&+3n_2HM(G_1)+3ReZG_3(G_1)+m_1^2(6m_2+m_1n_2)+m_1n_2^3.\\
(iv)F(G_1 \underline{\vee}_T G_2) \;=\;& 8F(G_1)+F(G_2)+3n_2^2M_1(G_1)+3m_1M_1(G_2)+Y(G_1)\\
&+3n_2HM(G_1)+3ReZG_3(G_1)+m_1^2(6m_2+m_1n_2)+m_1n_2^3.
\end{aligned}
$$

Theorem 3.5.4 31 *Let G_1 and G_2 be two vertex disjoint graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. Then*

$$
\begin{aligned}
(i)Y(G_1 \underline{\vee}_S G_2) \;=\;& (n_1+n_2-1)F(G_1)+F(G_2)+3m_1M_1(G_2)+6m_1^2m_2+m_1(n_2+2)^3\\
&+n_2m_1^3-Y(G_1)+Y(G_2)+m_1(2+n_2)^4+4m_1F(G_2)+6m_1^2M_1(G_2)\\
&+8m_2m_1^3+n_2m_1^4.\\
(ii)Y(G_1 \underline{\vee}_R G_2) \;=\;& (n_1+n_2-1)8F(G_1)+F(G_2)+3m_1M_1(G_2)+6m_1^2m_2+m_1(n_2+2)^3\\
&+n_2m_1^3-16Y(G_1)+Y(G_2)+4F(G_2)m_1+6M_1(G_2)m_1^2+8m_2m_1^3\\
&+n_2m_1^4+m_1(n_2+2)^4.\\
(iii)Y(G_1 \underline{\vee}_Q G_2) \;=\;& (n_1+n_2-1)F(G_1)+F(G_2)+3n_2^2M_1(G_1)+3m_1M_1(G_2)\\
&+Y(G_1)+3n_2HM(G_1)+3ReZG_3(G_1)+m_1^2(6m_2+m_1n_2)+m_1n_2^3\\
&-Y(G_1)+Y(G_2)+4F(G_2)m_1+6M_1(G_2)m_1^2+8m_2m_1^3+n_2m_1^4\\
&+\xi_5(G_1)+ReZ_4(G_1)+ReZ_4^2(G_1)+Y(G_1)n_2+12DD(G_1)n_2\\
&+6F(G_1)n_2^2+12M_2(G_1)n_2^2+4M_1(G_1)n_2^3+n_1n_2^4.\\
(iv)Y(G_1 \underline{\vee}_T G_2) \;=\;& (n_1+n_2-1)8F(G_1)+F(G_2)+3n_2^2M_1(G_1)+3m_1M_1(G_2)+Y(G_1)\\
&+3n_2HM(G_1)+3ReZG_3(G_1)+m_1^2(6m_2+m_1n_2)+m_1n_2^3\\
&-16Y(G_1)+Y(G_2)+4F(G_2)m_1+6M_1(G_2)m_1^2+8m_2m_1^3+n_2m_1^4\\
&+\xi_5(G_1)+ReZ_4(G_1)+ReZ_4^2(G_1)+Y(G_1)n_2+12DD(G_1)n_2\\
&+6F(G_1)n_2^2+12M_2(G_1)n_2^2+4M_1(G_1)n_2^3+n_1n_2^4.
\end{aligned}
$$

4. Conclusion

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In this paper, we have established some useful formula for Y −index and coindex of graphs based on the vertex and edge $\mathcal{F}$ −join of graphs where $F = \{S, R, Q, T\}$, and we have given some examples for these graphs. For future study, other topological indices for this graph operations can be computed.

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