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$convex_{IVIF}$ Sets in Interval-Valued Intuitionistic Fuzzy Topological Vector Spaces

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Abstract: The aim of this research work is to introduce the new concept of interval-valued intuitionistic fuzzy topological vector space(in brief IVIF-topological vector space). In this paper, the concept of IVIF-vector point, IVIF-Quasicoincidence were defined. In further, the relationship between IVIF-N_{QC}neighbourhood, IVIF-QCneighbourhood and its bases in IVIF-topological vector space discussed. Also introduced the concept of $convex_{IVIF}$set, Strongly $convex_{IVIF}$ set and Strictly $convex_{IVIF}$ set, in IVIF-topological vector spaces. Then it continued to the discussion about the properties and the relationship between these $convex_{IVIF}$ sets.
Keywords:IVIF-Quasicoincidence, IVIF-vector points, IVIF-topological vector space, IVIF-neighbourhood points, IVIF-neighbourhood base points, IVIF-N_{QC}neighbourhood, IVIF-QCneighbourhood, $convex_{IVIF}$ set, Strongly $convex_{IVIF}$ set, Strictly $convex_{IVIF}$ set.

1. Introduction:

The Fuzzy set concept was introduced by Zadeh[26]. The idea of interval-valued intuitionistic fuzzy set was introduced by Atanassov[1]. Then Atanassov and Gargov[2] developed the properties of interval-valued intuitionistic fuzzy set. Amal Kumar Adak and Manoranjan Bhowmik[4] introduced different types of interval cut-set of IVIFSs, also investigate some properties of those cut-set of IVIFSs. Francisco Gallego Lupianez[8] defined and studied the notion of quasi-coincidence for intuitionistic fuzzy points and obtain a characterization of continuity for maps between intuitionistic fuzzy topological spaces. Also Coker and Mustafa Demirici[7], introduced Quasi-coincidence and Pseudo-coincidence of intuitionistic fuzzy points. In further Coker[6] introduced the basic concepts of intuitionistic fuzzy topological spaces. In 2004, Kul Hur et al.,[10] introduced the fundamental concepts of intuitionistic fuzzy Q-neighbourhood, intuitionistic Q-first axiom of countability, intuitionistic fuzzy closure operator, intuitionistic fuzzy boundary point and intuitionistic fuzzy accumulation point and investigate some of their properties.

In 1977, Katsaras and Liu[9] apply the concept of a fuzzy set to the elementary theory of vector spaces and topological vector spaces. Topologically complete intuitionistic fuzzy metrizable spaces was introduced by Reza Saadati and Jin Han Park[16]. Topology of interval-valued intuitionistic fuzzy setconcept was introduced by Tapas Kumar Mondal and Samanta[21]. In 2014, Mohammed Jassim Mohammed and Ghufran Adeel Ataa[12] introduced and studied the concept of intuitionistic fuzzy topological vector spaces. Vijaya

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Balaji and Sivaramakrishnan[22] construct the cartesian product and homomorphism of interval-valued fuzzy linear space. In our previous work[19], we defined interval-valued intuitionistic fuzzy vector spaces and developed some of its properties. In further, we equip the concept of topology of interval-valued intuitionistic fuzzy sets to interval-valued intuitionistic fuzzy topological vector spaces.

Zadeh introduced the concept of convex fuzzy sets, which is an important kind of extension of classical convex sets from the view point of cut set. Following this, the theory and applications about convex fuzzy sets have been studied intensively by the following authors[5, 13,14, 15, 24, 25]. However, the corresponding research of convex intuitionistic fuzzy set is rather scarce, which restricts its application greatly. In 2018, Sangeetha Saha and Pradip Debnath[18], introduced new properties on φ-convex and φ-quasiconvex in intuitionistic fuzzy sets. Aggregation of convex intuitionistic fuzzy set was introduced by Susana Diaz et al.,[20]. X Pan[23] introduced a new concept of graded convex intuitionistic fuzzy set.

In this paper, section 2 gives some preliminary definitions which were used for section 3 and 4. In section 3, the concept of IVIF-Quasicoincidence in IVIF-vector points then continue it to IVIF-topological vector space and its neighbourhood points were discussed. The definition of$convex_{IVIF}$ set, strongly $convex_{IVIF}$ set, strictly $convex_{IVIF}$ set and their basic properties were given in section 4.

2. Preliminaries

This section gives some basic definitions of interval-valued intuitionistic fuzzy set which are need to further study.

Definition 2.1.[17]: An interval-valued fuzzy set (IVFS) A on the universe $U \neq \phi$ is given by$A = \{(u, A(u)): u \in U\}$, where $A(u) = [\underline{A}(u), \overline{A}(u)] \in L([0,1])$ being $L([0,1]) = \{x = [\underline{x}, \overline{x}]: [\underline{x}, \overline{x}] \in [0,1]^2$ and $\underline{x} \leq \overline{x}\}$. Obviously, $A(u)$ is the membership degree and $\underline{A}(u), \overline{A}(u)$ are the lower and the upper limits of the membership degree of $u \in U$. Let I be the set of all real numbers lying between 0 and 1. That is, $I = \{x: 0 \leq x \leq 1\}$. Also let $D[0,1]$ be the interval$[0,1]$ which can be written as $D = \{[a, b]: a \leq b;\ for\ all\ a, b \in I\}$.

Definition 2.2.[17]: An interval-valued fuzzy vector is an n-tuple of elements from an interval-valued fuzzy algebra. That is, an IVFV is of the form $(x_1, x_2, \dots x_n)$, where each element $x_i \in F, i = 1,2, \dots n$.

Definition 2.3.[17]: An interval-valued fuzzy vector space(IVFVS) is a pair $(E, A(x))$, where E is a vector space in crisp sense and $A: E \to D[0,1]$ with the property, that for all $a, b \in F$ and $x, y \in E$, we have$\underline{A}(ax + by) \geq \underline{A}(x) \wedge \underline{A}(y)$ and $\overline{A}(ax + by) \geq \overline{A}(x) \wedge \overline{A}(y)$.

Definition 2.4.[1]: Let $D[0,1]$ be the set of closed subintervals of the interval $[0,1]$ and $X(\neq \phi)$ be a given set. An interval-valued intuitionistic fuzzy set in X is defined as, $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \ /\ x \in X\}$, where $\mu_A: X \to D[0,1], \nu_A: X \to D[0,1]$with the condition $0 \leq \sup(\mu_A(x)) + \sup(\nu_A(x)) \leq 1$ for any $x \in X$.The intervals $\mu_A(x)$ and $\nu_A(x)$ denotes the degree of belongingness and the degree of nonbelongingness of the element x to the set

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A. Thus for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$are closed intervals and their lower and upper end points are denoted by $\mu_{A_L}(x), \mu_{A_U}(x)$, $\nu_{A_L}(x)$ and $\nu_{A_U}(x)$. We can denote it by:

$$A = \{\langle x, [\mu_{A_L}(x), \mu_{A_U}(x)], [\nu_{A_L}(x), \nu_{A_U}(x)]\rangle \,/\, x \in X\},$$

Where $0 \leq \mu_{A_U}(x) + \nu_{A_U}(x) \leq 1, \mu_{A_L}(x) \geq 0, \nu_{A_L}(x) \geq 0$.

Note2.5.[1]: For each element x, we can compute the unknown degree(hesitancy degree) of an intuitionistic fuzzy interval of $x \in X$ in A, defined as follows:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = [1 - \mu_{A_U}(x) - \nu_{A_U}(x), 1 - \mu_{A_L}(x) - \nu_{A_L}(x)]$$

Especially, if $\mu_A(x) = \mu_{A_U}(x) = \mu_{A_L}(x)$ and $\nu_A(x) = \nu_{A_U}(x) = \nu_{A_L}(x)$, then the given IVIFS A is reduced to an ordinary intuitionistic fuzzy set.

Definition 2.6. [2]: For two IVIFSs, $A = \{\langle x, [\mu_{A_L}(x), \mu_{A_U}(x)], [\nu_{A_L}(x), \nu_{A_U}(x)]\rangle \,/\, x \in X\}$, and $B = \{\langle x, [\mu_{B_L}(x), \mu_{B_U}(x)], [\nu_{B_L}(x), \nu_{B_U}(x)]\rangle \,/\, x \in X\}$, the following two relations are defined:

i. $A \subseteq B$ if and only if
   a) $\mu_{A_U}(x) \leq \mu_{B_U}(x)$,
   b) $\mu_{A_L}(x) \leq \mu_{B_L}(x)$,
   c) $\nu_{A_U}(x) \geq \nu_{B_U}$(x),
   d) $\nu_{A_L}(x) \geq \nu_{B_L}(x)$ for any $x \in X$.

ii. $A = B$ if and only if
   a) $\mu_{A_U}(x) = \mu_{B_U}(x)$,
   b) $\mu_{A_L}(x) = \mu_{B_L}(x)$,
   c) $\nu_{A_U}(x) = \nu_{B_U}$(x),
   d) $\nu_{A_L}(x) = \nu_{B_L}(x)$ for any $x \in X$.

Definition 2.7.[21]: A topological space of IIF sets is a pair (X, τ), where X is a nonempty set and τ is a family of IIF sets on X, satisfying the following three axioms:

i. $\tilde{0}, \tilde{1} \in \tau$;

ii. $A, B \in \tau$, implies $A \cap B \in \tau$;

iii. $A_i \in \tau, i \in \Delta$ implies$\bigcup_{i \in \Delta} A_i \in \tau$.

where τ is called a topology of IIF sets on X. Every member of τ is called open. And $B \in IIF(X)$ is said to be closed in (X, τ) iff $B^c \in \tau$. As in ordinary topologies, the indiscrete topology of IIF sets contains only $\tilde{0}$ and $\tilde{1}$, while the discrete topology of IIF sets contains all IIF sets. These two topologies are respectively, denoted by τ^0 and τ^1. A topology τ_1 is said to be weaker(or coarser) than a topology τ_2 iff $\tau_1 \subseteq \tau_2$. In that case τ_2 is said to be stronger (or finer) than τ_1.

Definition 2.8.[21]: Let (X, τ) be a topological space of IIF sets. A subcollection $\mathcal{B}$ of τ is said to be a base for τ if every member of τ can be expressed as a union of member of $\mathcal{B}$.

Definition 2.9.[21]: Let (X, τ) be a topological space of IIF sets. A subcollection $\mathcal{G}$ of τ is said to be a subbase for τ if the family of all finite intersections of members of $\mathcal{G}$ forms a base for τ.

Definition 2.10.[21]: An IIF set A in a topological space of IIF sets (X, τ) is said to be a neighbourhood(nbd) of an IIF points P_x iff there exists $O \in \tau$ such that $P_x \in O \subset A$.

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Definition 2.11.[21]: Let A be an IIF set in a topological space of IIF sets (X,τ). Then an IIF point P_x is said to be an interior point of A if and only if A is a neighbourhood of P_x. The union of all interior points of A is called the interior of A and is denoted by $int(A)$.

Definition 2.12.[21]: Let (X,τ) be a topological space of IIF sets and $A \in II(X)$. Closure of A is defined as $\bigcap\{B \in II(X); B\ is\ closed\ and\ A \subset B\}$ and denote it by $cl(A)$.

Definition 2.13.[19]: Let $\tilde{V}$ be an interval-valued intuitionistic fuzzy vector space and it is a pair, $(V, \langle[\alpha_{\mu_L}(x), \alpha_{\mu_U}(x)], [\alpha_{\nu_L}(x), \alpha_{\nu_U}(x)]\rangle)$ and $\alpha_{\mu_U}: \tilde{V} \to D[0,1], \alpha_{\nu_U}: \tilde{V} \to D[0,1], \alpha_{\mu_L} \geq 0$ and $\alpha_{\nu_L} \geq 0$ with the property that for all $\alpha, \beta \in \tilde{V}$ and $x, y \in F$ then

i. $[(\alpha_{\mu_L} + \beta_{\mu_L}), (\alpha_{\mu_U} + \beta_{\mu_U})] \in \tilde{V}$and $[(\alpha_{\nu_L} + \beta_{\nu_L}), (\alpha_{\nu_U} + \beta_{\nu_U})] \in \tilde{V}$

ii. $\alpha x \in \tilde{V}, i.e., \{\langle[(\alpha_{\mu_L} \wedge x_{\mu_L}), (\alpha_{\mu_U} \wedge x_{\mu_U})], [((1-\alpha_{\nu_L}) \vee x_{\nu_L}), ((1-\alpha_{\nu_U}) \vee x_{\nu_U})]\rangle \in \tilde{V}$

Where, $\alpha_{\mu_L} + \beta_{\mu_L} = \alpha_{\mu_L} \vee \beta_{\mu_L}$; $\alpha_{\mu_U} + \beta_{\mu_U} = \alpha_{\mu_U} \vee \beta_{\mu_U}$;$\alpha_{\nu_L} + \beta_{\nu_L} = \alpha_{\nu_L} \wedge \beta_{\nu_L}$ and $\alpha_{\nu_U} + \beta_{\nu_U} = \alpha_{\nu_U} \wedge \beta_{\nu_U}$.

3. Interval-Valued Intuitionistic Fuzzy Topological Vector Space

In this section, IVIF-Quasicoincidence of IVIF set, IVIF-topological vector spaces are defined and discusses the properties of IVIF-QC neighbourhood, IVIF-N_{QC} neighbourhood. Throughout the following sections, $\phi = \langle[0,0],[1,1]\rangle$ is the zero element and $I = \langle[1,1],[0,0]\rangle$ is the identity element.

Definition 3.1.: Let $\tilde{V}$be a nonempty set of IVIF-vectors. An IVIF-vector point, denoted by $P_x = \{\langle x, [\mu_A^L, \mu_A^U], [\nu_A^L, \nu_A^U]\rangle\}$ is in IVIF-set P such that there is an $x \in \tilde{V}$ and $y \in P$ satisfying:

$$[\mu_A^L, \mu_A^U](y) = \begin{cases} [\mu^L, \mu^U] & if\ x = y \\ \phi & Otherwise \end{cases}$$

$$[\nu_A^L, \nu_A^U](y) = \begin{cases} [\nu^L, \nu^U] & if\ x = y \\ I & Otherwise \end{cases}$$

Where $x \in \tilde{V}$ is a fixed IVIF-vector point. The set of all IVIF-vector points P_x is denoted by $P_t(\tilde{V})$.

Definition 3.2.: An IVIF-vector point P_x is said to belongs to an IVIF-set A if $[\mu^L, \mu^U](x) \leq [\mu_A^L, \mu_A^U](x)$ and $[\nu^L, \nu^U](x) \geq [\nu_A^L, \nu_A^U](x)$. It is denoted by $P_x \in A$.

Definition 3.3.: Let $A = \{\langle x, [\mu_A^L, \mu_A^U](x), [\nu_A^L, \nu_A^U](x)\rangle \,/\, x \in \tilde{V}\}$ and $B = \{\langle x, [\mu_B^L, \mu_B^U](x), [\nu_B^L, \nu_B^U](x)\rangle \,/\, x \in \tilde{V}\}$ be two IVIF sets in $\tilde{V}$. If there exists x in $\tilde{V}$ such that $[\mu_A^L, \mu_A^U](x) > [\nu_B^L, \nu_B^U](x)$ or $[\nu_A^L, \nu_A^U](x) < [\mu_B^L, \mu_B^U](x)$.

That is $\mu_A^L(x) > \nu_B^L(x)$ and $\mu_A^U(x) > \nu_B^U(x)$ or $\nu_A^L(x) < \mu_B^L(x)$ and $\nu_A^U(x) < \mu_B^U(x)$. Then A is said to be IVIF-Quasicoincident with B and is denoted by $A_q B$. Otherwise A is not IVIF-Quasicoincident with B and it is denoted by $A\ \not{q} B$.

Lemma 3.4.:$P(\langle[\mu^L, \mu^U], [\nu^L, \nu^U]\rangle) \in A$ if and only if $P(\langle[\mu^L, \mu^U], [\nu^L, \nu^U]\rangle) \not{q}\ A^c$.

Proof: The result of this Lemma follows from the Definitions 3.1., 3.2., and 3.3.

Theorem 3.5.: Let A and B be two IVIF-sets in $\tilde{V}$, then

i. $A \not{q} B$ if and only if $A \subseteq B^c$

ii. AqB if and only if $A \not\subseteq B^c$.

Proof: Let $A = \{\langle x, [\mu_A^L, \mu_A^U](x), [\nu_A^L, \nu_A^U](x)\rangle / x \in \tilde{V}\}$ and $B = \{\langle x, [\mu_B^L, \mu_B^U](x), [\nu_B^L, \nu_B^U](x)\rangle / x \in \tilde{V}\}$ be two IVIF-sets in $\tilde{V}$. Then there exists $x \in \tilde{V}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p.2569-2577
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such that by Lemma 3.4., $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in A$if and only if $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \not q\, A^c$ and $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in B$ if and only if $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \not q\, B^c$.

To Prove(i): Assume that $A \not q B$. This implies that $[\mu_A^L,\mu_A^U] < [\nu_B^L,\nu_B^U]$ or $[\nu_A^L,\nu_A^U] > [\mu_B^L,\mu_B^U]$. Therefore $A \nsubseteq B$. Hence $A \subseteq B^c$.

In this manner, we can prove the converse part.

To Prove(ii): Assume that AqB. This implies that $[\mu_A^L,\mu_A^U] > [\nu_B^L,\nu_B^U]$ or $[\nu_A^L,\nu_A^U] < [\mu_B^L,\mu_B^U]$. Therefore $A \subseteq B$. Hence $A \nsubseteq B^c$.

Proposition 3.6.: Let A,B be an IVIF sets and $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in P_t(\tilde{V})$. For $A \subseteq B$ if and only if $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in A$ then $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in B$.

Proof: Suppose that $A \subseteq B$. Assume that $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in A$ for all $x \in \tilde{V}$, $[\mu^L,\mu^U](x) < [\mu_A^L,\mu_A^U](x)$ and $[\nu^L,\nu^U](x) > [\nu_A^L,\nu_A^U](x)$. Since $A \subseteq B$, we get $[\mu_A^L,\mu_A^U](x) < [\mu_B^L,\mu_B^U](x)$ and $[\nu_A^L,\nu_A^U](x) > [\nu_B^L,\nu_B^U](x)$. This implies that $[\mu^L,\mu^U](x) < [\mu_B^L,\mu_B^U](x)$ and $[\nu^L,\nu^U](x) > [\nu_B^L,\nu_B^U](x)$. Hence $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in B$.

Conversely, suppose that $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in A$ then $P(\langle[\mu^L,\mu^U],[\nu^L,\nu^U]\rangle) \in B$. This implies $[\mu^L,\mu^U](x) < [\mu_A^L,\mu_A^U](x)$ and $[\nu^L,\nu^U](x) > [\nu_A^L,\nu_A^U](x)$. This implies that $[\mu^L,\mu^U](x) < [\mu_B^L,\mu_B^U](x)$ and $[\nu^L,\nu^U](x) > [\nu_B^L,\nu_B^U](x)$. Therefore,$[\mu_A^L,\mu_A^U](x) < [\mu_B^L,\mu_B^U](x)$ and $[\nu_A^L,\nu_A^U](x) > [\nu_B^L,\nu_B^U](x)$ for all $x \in \tilde{V}$. Hence $A \subseteq B$.

Definition 3.7.: Let $\tilde{\tau}$ be an IVIF-topology on the pair $(\tilde{V},\tilde{\tau})$ is called an IVIF-topological vector space if the IVIF-sets on $\tilde{V}$ satisfies the following two operations:

i) $+ : \tilde{V} \times \tilde{V} \to \tilde{V}$ by $(\alpha,\beta) = \alpha + \beta$;

ii) $\cdot : F \times \tilde{V} \to \tilde{V}$ by $(k,\alpha) = k\alpha$.

Where $\alpha = ([\mu_\alpha^L,\mu_\alpha^U],[\nu_\alpha^L,\nu_\alpha^U])$ and $\beta = ([\mu_\beta^L,\mu_\beta^U],[\nu_\beta^L,\nu_\beta^U])$. These two operations are IVIF-continuous, F has usual IVIF-topology and$\tilde{V} \times \tilde{V}$ and $F \times \tilde{V}$ are the IVIF-product topologies.

Definition 3.8.: If A and B are IVIF-sets in IVIF-topological vector space $\tilde{V}$ over$F$ and $k \in F$, then$A + B$ in$\tilde{V}$is defined as,

$$(A+B) = \langle[\mu_{(A+B)}^L,\mu_{(A+B)}^U],[\nu_{(A+B)}^L,\nu_{(A+B)}^U]\rangle$$

That is, $[\mu_{(A+B)}^L,\mu_{(A+B)}^U] = [\mu_A^L \vee \mu_B^L, \mu_A^U \vee \mu_B^U]$ and$[\nu_{(A+B)}^L,\nu_{(A+B)}^U] = [\nu_A^L \wedge \nu_B^L, \nu_A^U \wedge \nu_B^U]$.

Definition 3.9.: If $A$is an IVIF-set in IVIF-topological vector space$\tilde{V}$ over F and $k \in F$ then kA in $\tilde{V}$ is defined as follows:

$$kA = \langle k[\mu_A^L,\mu_A^U], k[\nu_A^L,\nu_A^U]\rangle$$

$$k[\mu_A^L,\mu_A^U](\alpha) = \begin{cases} ([\mu_k^L \wedge \mu_A^L](\alpha),[\mu_k^U \wedge \mu_A^U](\alpha)) & if\ k \neq \phi\ for\ all\ \alpha \in \tilde{V} \\ \phi & if\ k = \phi, \alpha \neq \phi \end{cases}$$

$$k[\nu_A^L,\nu_A^U](\alpha) = \begin{cases} ([(1-\nu_k^L) \vee \nu_A^L](\alpha),[(1-\nu_k^U) \vee \nu_A^U](\alpha)) & if\ k \neq I\ for\ all\ \alpha \in \tilde{V} \\ I & if\ k = I, \alpha \neq \phi \end{cases}$$

Definition 3.10.: An IVIF-subset A on$\tilde{V}$ is said to be an IVIF-neighbourhood points of $x \in \tilde{V}$, if there is $O \in \tilde{\tau}$ such that$[\mu^L,\mu^U](O(t)) \leq [\mu^L,\mu^U](A(t))$ and $[\nu^L,\nu^U](O(t)) \geq [\nu^L,\nu^U](A(t))$, for all $t \in \tilde{V}$. The set of all IVIF-neighbourhood points of x on $\tilde{V}$ is denoted by $\mathfrak{B}_{\tilde{V}}$.

Definition 3.11.: A subcollection $\mathfrak{B}$ of an IVIF-neighbourhood points of x is said to be an IVIF-base if$[\mu^L,\mu^U] \in [\phi,[\mu_A^L,\mu_A^U])(x)$ and $[\nu^L,\nu^U] \in ([\nu_A^L,\nu_A^U](x), I]$.

JOURNAL OF ALGEBRAIC STATISTICS
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For $A \in \tilde{V}$ there exists $B \in \mathfrak{B}$, such that $[\mu_B^L, \mu_B^U](t) \leq [\mu_A^L, \mu_A^U](t)$ and $[\nu_B^L, \nu_B^U](t) \geq [\nu_A^L, \nu_A^U](t), t \in \tilde{V}, [\mu_B^L, \mu_B^U](x) > [\mu^L, \mu^U]$ and $[\nu_B^L, \nu_B^U](x) < [\nu^L, \nu^U]$.

Definition3.12.: Let A be an IVIF-set. If there exists $O \in \tilde{\tau}$ such that $P_x \notin O^c$ and $A \subseteq O^c$ then the subset of $\tilde{V}$ is called an IVIF-N_{QC}neighbourhood of P_x.

Definition3.13.: Let A be an IVIF set of $\tilde{V}$ is said to be an IVIF-QC neighbourhood of P_x if there exist $O \in \tilde{\tau}$ such that $P_x q O \subseteq A$. The family of all IVIF-QC neighbourhood of P_x, is denoted by $QC_N(P_x)$.

Definition 3.14.: For each $A \in QC_N(P_x)$, there exists $B \in P_t(x)$ such that$[\mu_B^L, \mu_B^U](t) \leq [\mu_A^L, \mu_A^U](x)$ and$[\nu_B^L, \nu_B^U](t) \geq [\nu_A^L, \nu_A^U](x)$; $x, t \in \tilde{V}$, then$QC_N(P_x)$ is said to be an IVIF-QC_N-base of P_x.

Definition 3.15.: An IVIF-set A on $(\tilde{V}, \tilde{\tau})$ is said to be an IVIF-neighbourhood of ϕ(in brief N_ϕ) if there is an $B \in \tilde{\tau}$ such that $N_\phi q B \subseteq A$.

Theorem 3.16.: Let A be an IVIF set on $\tilde{V}$. Then A is an IVIF-QC neighbourhood ofP_x if and only if A^c is an IVIF-N_{QC} neighbourhood of P_x.

Proof: Suppose that A is an IVIF-QC neighbourhood of P_x. Then there is an $O \in \tilde{\tau}$ such that $P_x q O \subseteq A$. We have,$P_x q O$ implies that $P_x \subseteq O$. And $P_x \notin O^c$. Since $O \subset A$. This implies $A^c \subseteq O^c$. From the above two properties, A^c is an IVIF-N_{QC} neighbourhood of P_x.

Conversely, assume that A^c is an IVIF-N_{QC}neighbourhood of P_x. Then there is an $O \in \tilde{\tau}$ such that $P_x \notin O^c$ and $A^c \subseteq O^c$. This implies that $P_x q O \subseteq A$. Since $A^c \subseteq O^c$ implies $O \subseteq A$. Thus there is an $O \in \tilde{\tau}$ such that $P_x q O \subseteq A$. Hence A is an IVIF-QC neighbourhood of P_x.

Theorem 3.17.:A is an IVIF-QC neighbourhood of N_ϕ if and only if A is an IVIF-neighbourhood of ϕ.

Proof: Assume that A be an IVIF-neighbourhood of ϕ. There is an $O \in \tilde{\tau}$ such that $N_\phi q O \subseteq A$. This implies that, A is an IVIF-QC neighbourhood of N_ϕ.

Conversely assume that, A is an IVIF-QC neighbourhood of N_ϕ. By Definition 3.15., there exists an $O \in \tilde{\tau}$ such that $N_\phi q O \subseteq A$. This implies that, at the point of N_ϕ, A is an IVIF-QC neighbourhood of N_ϕ. Therefore, A is an IVIF-neighbourhood of ϕ.

Remark 3.18.: From Theorem 3.17., we can say: Let A be an IVIF-neighbourhood of ϕ if and only if $A^c$is an IVIF-N_{QC}neighbourhood of N_ϕ.

4. $convex_{IVIF}$ sets in IVIF-topological vector spaces

In this section, the discussion is about the definition of $convex_{IVIF}$ sets and its properties.

Definition 4.1.: An interval-valued intuitionistic fuzzy set A on $\tilde{V}$ is said to be a $convex_{IVIF}$ set if $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ for all $x, y \in \tilde{V}$ and $\alpha \in D[0,1]$.

Definition 4.2.: An interval-valued intuitionistic fuzzy set A on $\tilde{V}$ is said to be a strongly $convex_{IVIF}$ set if $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ for all $x, y \in \tilde{V}, x \neq y$ and $\alpha \in D[0,1]$.

Definition 4.3.: An interval-valued intuitionistic fuzzy set A on $\tilde{V}$ is said to be a strictly $convex_{IVIF}$ set if $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) > [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x -$

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$y) + y) < [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ for all $x, y \in \tilde{V}$, $[\mu_A^L, \mu_A^U](x) \neq [\mu_A^L, \mu_A^U](y), [\nu_A^L, \nu_A^U](x) \neq [\nu_A^L, \nu_A^U](y)$and $\alpha \in D[0,1]$.

Proposition 4.4.: Let A be an IVIF-set on $\tilde{V}$. If there exists $\alpha \in D[0,1]$,for every $x, y \in \tilde{V}$, then the following statements are obvious:

i) Let A be a strictly $convex_{IVIF}$ set on $\tilde{V}$. If there exists $\alpha \in D[0,1]$, for every $x, y \in \tilde{V}$ and $[\mu_A^L, \mu_A^U](x) \neq [\mu_A^L, \mu_A^U](y), [\nu_A^L, \nu_A^U](x) \neq [\nu_A^L, \nu_A^U](y)$ such that $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) > [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) < [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ then A is a $convex_{IVIF}$ set on $\tilde{V}$.

ii) Let A be a strongly $convex_{IVIF}$ set on $\tilde{V}$. If there exists $\alpha \in D[0,1]$, for every pair of distinct points $x, y \in \tilde{V}$ implies that, $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$, then A is a $convex_{IVIF}$ set on $\tilde{V}$.

iii) Let A be a strictly$convex_{IVIF}$ set on $\tilde{V}$. If there exists $\alpha \in D[0,1]$ such that for every pair of distinct points $x, y \in \tilde{V}$, we have $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) > [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) < [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ then A is a strongly $convex_{IVIF}$ set on $\tilde{V}$.

Proof: By Definitions 4.1, 4.2 and 4.3, the proof of the above statements is obviously true.

Note 4.5.: Let A be a $convex_{IVIF}$ set on $\tilde{V}$. If there exists $\alpha \in D[0,1]$,for every $x, y \in \tilde{V}$ such that $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ then A is not a strictly $convex_{IVIF}$ set on $\tilde{V}$.

Note 4.6.: Let A be a $convex_{IVIF}$set on $\tilde{V}$. There exists $\alpha \in D[0,1]$, for every $x, y \in \tilde{V}$, $[\mu_A^L, \mu_A^U](x) \neq [\mu_A^L, \mu_A^U](y), [\nu_A^L, \nu_A^U](x) \neq [\nu_A^L, \nu_A^U](y)$, such that $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) > [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) < [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$, then A is a strictly $convex_{IVIF}$ set on $\tilde{V}$.

Note 4.7.: Let A be a strong $convex_{IVIF}$ set on a IVIF topological vector space $\tilde{V}$. There exists $\alpha \in D[0,1]$ such that for every pair of distinct points $x, y \in \tilde{V}$, we have $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$ then A is not a strictly $convex_{IVIF}$ set on $\tilde{V}$.

Proposition 4.8.: If A is a $convex_{IVIF}$ set and IVIF-QC neighbourhood of N_ϕ, then $int(A)$ is also $convex_{IVIF}$ set and IVIF-QC neighbourhood of N_ϕ.

Proof: Assume that an IVIF set A is IVIF-QC neighbourhood of N_ϕ and $convex_{IVIF}$set. By Definitions, there exists $O \in \tilde{\tau}$ such that $N_\phi qO \subseteq A$. Since $O \in \tilde{\tau}$. This implies, O is an IVIF-open set. So, $int(O) \subseteq int(A)$. Since $O \subseteq A$. Combining the above properties, we get, $N_\phi qO \subseteq int(A)$. Therefore $int(A)$ is an IVIF-QC neighbourhood of N_ϕ.

Next, assume that, A is a $convex_{IVIF}$ set. This implies $[\mu_A^L, \mu_A^U](\alpha(x-y)+y) \geq [\mu_A^L, \mu_A^U](x) \wedge [\mu_A^L, \mu_A^U](y)$ and $[\nu_A^L, \nu_A^U](\alpha(x-y)+y) \leq [\nu_A^L, \nu_A^U](x) \vee [\nu_A^L, \nu_A^U](y)$, for every $x, y \in \tilde{V}$ and $\alpha \in D[0,1]$. So, $int([\mu_A^L, \mu_A^U](\alpha(x-y)+y)) \geq int([\mu_A^L, \mu_A^U](x)) \wedge int([\mu_A^L, \mu_A^U](y))$ and $int([\nu_A^L, \nu_A^U](\alpha(x-y)+y)) \leq int([\nu_A^L, \nu_A^U](x)) \vee int([\nu_A^L, \nu_A^U](y))$, for all $x, y \in \tilde{V}$ and $\alpha \in D[0,1]$. Hence $int(A)$ is also $convex_{IVIF}$ set.

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