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To scrutinizes finite source queuing models with pentagonal fuzzy numbers using centroid of centroids technique under imprecise environment

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ABSTRACT
In this article, we looked at performing a pentagonal fuzzy number using a single server and multi server system in a finite source queue paradigm. The approach is built on a first-come, first served queue. The pentagonal fuzzy number is used to depict the arrival and service times. To convert the times of arrival and service into a crisp value using centroid of centroids method for pentagonal fuzzy numbers. This strategy is quite effective rapidly converted into a crisp value from a pentagonal fuzzy numbers. After that, converted crisp values are applied to the standard formulas for the finite source queue model. This method is exemplified using numerical examples.

Keywords Fuzzy set · Fuzzy number · pentagonal fuzzy number · centroid of centroids ranking · finite source fuzzy queue model.

1 Introduction

Queuing theory is an important facet of operational research waiting line applications are highly important in everyday life, as well as in computer programming, networks, medical fields, financial sectors, contact centers, telecommunications, manufacturing and production.

Customers come to a queue in various conditions, and queues with continuous service are common in everyday life. However, not every service is available right away. Typically, clients in such a case, exit the service area. After a predetermined amount of time has elapsed, they will acquire what they require, after finishing their services.

However, if the service provider does not predict the precise time, he or she may not be able to provide the service on time to the client in some instances. To avert such an environment, finite-source queue models are crucial. We may quickly discover problems in a queue and offer service to clients at a certain time if you compute the customer queue and service time for them in a finite source queue. We must transmit the service to the client on promptly, especially during peak service-hours on a single server queue only. Here we explore to describe the consequences on a multiple server queuing model also.

To analyze the customer insights and service time, we deploy finite – source queues. The service remains unaffected by departing customers, which is a unique characteristic of this approach. The purpose of this article is to use finite source queue models to determine the arrival and service time probability of customers in a queue, as well as the average waiting time of customers in a single and multiple server queuing model using the centroid of centroids ranking approach.

Various ranking approaches have been used by many researchers to assess the performance of fuzzy queues. One of the most well-known ranking techniques is robust ranking technique. Although, distance-based techniques are the best methods of ranking approach. That is the area between the centroid and the original points, the circumcenter of centroids and the ranking algorithms for the centroid of centroids.

The standard formulas of single and multiple channel finite source queue models may also be evaluated using the centroid of centroids ranking approach. The exact crisp values of the queuing models may be retrieved using this specific technique.

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Literature Review

Black, M., suggested reasoning with loose concepts in 1963. Jain, R., introduced a procedure for multiple-aspect decision using fuzzy sets and he studied tolerance analysis using fuzzy sets in 1977. Dubois, D., and Prade, H., were proposed operations on Fuzzy Numbers in 1978. Dijkman, J.G.,Haeringen, H. van., and Lange, S.J. de., were discussed the concept of Fuzzy Numbers in 1983. Atanassov, K.T., analyzed intuitionistic fuzzy sets in 1986. Hellendoorn, H., depicted fuzzy numbers and approximate reasoning in 1991.

Shawky., investigated the machine interference model, M/M/C/K/N with balking, reneging and spares in 2000.Hanss, M., illustrated the applied fuzzy arithmetic: an introduction with engineering applications in 2005. Bansal,A., proposed trapezoidal fuzzy numbers (a,b,c,d): arithmetic behavior in 2011.Banerjee, S., and Roy, T.K., discussed about arithmetic operations on generalized trapezoidal fuzzy number and its applications in 2012.Gani,A.N., and Assarudeen,A.N.M., suggested a new operation on triangular fuzzy number for solving fuzzy linear programming problem in 2012. Bekheet, S., studied polygon fuzzy numbers in 2013.

Helen, R., and Uma, G., presented a new operation and ranking on pentagon fuzzy number in 2015. Pathinathan. T., Ponnivalavan. K. and Mike Dison. E., scrutinized different types of fuzzy numbers and certain properties in 2015.Saima Jabee., and Vandana Malviya., were explored Performance Measures of M/M/C/N Queuing Model in 2015.R. Srinivasan, T. Nakkeeran, K. Renganathan, and V. Vijayan., were examined the performance of pentagonal fuzzy numbers in finite source queue models using Pascal's triangular graded mean in 2020..

In this research article, we explored finite source queue models to determine the arrival and service time probability of customers in a queue, as well as the average waiting time of customers in a single and multiple server queuing models using the centroid of centroids ranking approach to scrutinizes the analogy between the generalized pentagonal fuzzy number and generalized intuitionistic pentagonal fuzzy number.

2 Preliminaries

2.1 Fuzzy set

If x belongs to the classical set $\tilde{A}$ in the pair $\left(x, \mu_{\tilde{A}}(x)\right)$ and the second set $\mu_{\tilde{A}}(x)$ belongs to the interval [0,1], then the set $\tilde{A}$ is called a fuzzy set. A membership function $\mu_{\tilde{A}}(x)$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in \tilde{A}, \mu_{\tilde{A}}(x) \in [0,1]\}$.

2.2 Fuzzy number

If set $\tilde{A}$ in $\tilde{R} \rightarrow [0,1]$ with a membership function is called a fuzzy number,

1) $\tilde{A}$ is normal.
2) $\tilde{A}$ is fuzzy concave .
3) $\mu_{\tilde{A}}$ is upper semi continuous.
4) Supp $\tilde{A}$ is bounded.

2.3 Pentagonal fuzzy number

A pentagonal fuzzy number $\tilde{A}$ is a fuzzy subset of real line *R*, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

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(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]
(ii) $\mu_{\tilde{A}}(x)=0,$ where $-\infty < x \le a_1$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $a_1 \le x \le a_2$
(iv) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $a_2 \le x \le a_3$
(v) $\mu_{\tilde{A}}(x)=1,$ where $x = a_3$
(vi) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $a_3 \le x \le a_4$
(vii) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $a_4 \le x \le a_5$
(viii) $\mu_{\tilde{A}}(x)=0,$ where $a_5 \le x \le \infty$

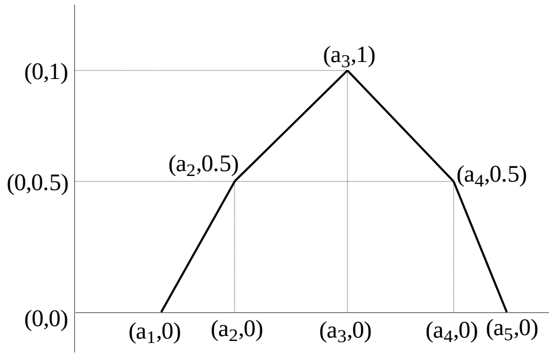

Fig. 1 – Pentagonal fuzzy number

Pentagonal fuzzy number is defined as $\tilde{A}=(a_1,a_2,a_3,a_4,a_5)$, where all a_1,a_2,a_3,a_4 and a_5 are real numbers and its membership function is given below.

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & ; \text{ for } x < a_1; \\ \frac{1}{2}\left(\frac{(x-a_1)}{(a_2-a_1)}\right) & ; \text{ for } a_1 \le x \le a_2; \\ \frac{1}{2}+\frac{1}{2}\left(\frac{(x-a_2)}{(a_3-a_2)}\right) & ; \text{ for } a_2 \le x \le a_3; \\ 1 & ; \text{ for } x = a_3, \\ \frac{1}{2}+\frac{1}{2}\left(\frac{(a_4-x)}{(a_4-a_3)}\right) & ; \text{ for } a_3 \le x \le a_4, \\ \frac{1}{2}\left(\frac{(a_5-x)}{(a_5-a_4)}\right) & ; \text{ for } a_4 \le x \le a_5, \\ 0 & ; \text{ for } x > a_5 \end{cases}$$

Conditions on PFN

1) $\mu_{\tilde{A}}(x)$ in interval [0,1] is a continuous function.
2) $\mu_{\tilde{A}}(x)$ is strictly non-decreasing continuous function at the $[a_1, a_2]$ and $[a_2, a_3]$ intervals.
3) $\mu_{\tilde{A}}(x)$ is strictly non-increasing continuous function at the $[a_3, a_4]$ and $[a_4, a_5]$ intervals.

2.4 Non-normalized (Generalized) pentagonal fuzzy number

The generalized pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5; w)$ is a fuzzy subset of real line *R*, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]

(ii) $\mu_{\tilde{A}}(x) = 0,$ where $-\infty < x \le a_1$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $a_1 \le x \le a_2$

(iv) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $a_2 \le x \le a_3$

(v) $\mu_{\tilde{A}}(x) = w,$ where $x = a_3$

(vi) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $a_3 \le x \le a_4$

(vii) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $a_4 \le x \le a_5$

(viii) $\mu_{\tilde{A}}(x) = 0,$ where $a_5 \le x \le \infty$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \quad x \le a_1 \\ \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & ; \quad a_1 \le x \le a_2 \\ \frac{1}{2} + \left(w - \frac{1}{2}\right)\left(\frac{x - a_2}{a_3 - a_2}\right) & ; \quad a_2 \le x \le a_3 \\ w & ; \quad x = a_3 \\ \frac{1}{2} + \left(w - \frac{1}{2}\right)\left(\frac{a_4 - x}{a_4 - a_3}\right) & ; \quad a_3 \le x \le a_4 \\ \frac{1}{2}\left(\frac{a_5 - x}{a_5 - a_4}\right) & ; \quad a_4 \le x \le a_5 \\ 0 & ; \quad x \ge a_5 \end{cases}$$

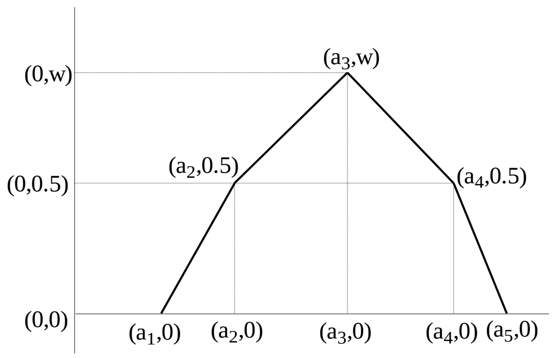

Fig. 2 – Generalized pentagonal fuzzy number with α cut = 0.5

2.5 Centroid of a pentagonal fuzzy number

The centroid of a pentagonal fuzzy number is determined, and it is distinguished from non-normalized pentagonal fuzzy numbers. We propose a simpler method to arrive at a compact formula for centroid of a pentagonal fuzzy number, as Helen.R, and Uma.G, has derived centroid of a pentagonal fuzzy number. The centroid of a pentagonal is presumed to be the pentagon's balance point (Figure 1). Divide pentagon into three plane figures. These three plane figures are a triangle in which (a_1, 0), (b_1, 0.5) and (c_1, 0) are the three vertices, a rhombus in which (b_1, 0.5), (c_1, 0), (d_1, 0.5) and (c_1, 0.5) represents the corner vertices and a triangle with (c_1, 0), (d_1, 0.5) and (e_1, 0) represents the vertices. Let the centroids of the three plane figures be G_1, G_2 and G_3 respectively and it is given by:

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$G_1 = \left(\frac{a_1+b_1+c_1}{3}, \frac{1}{6}\right)$; $G_2 = \left(C_1, \frac{1}{2}\right)$ and

$G_3 = \left(\frac{c_1+d_1+e_1}{3}, \frac{1}{6}\right)$

The following figure (Figure 3) shows the construction part of the centroid of a pentagonal fuzzy number with G_1, G_2 and G_3, the three centroids for the normal pentagonal fuzzy number.

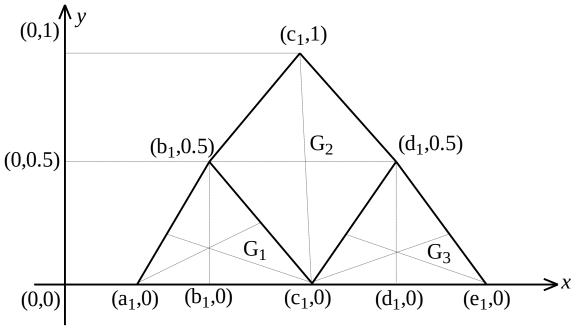

Fig. 3 – Construction of pentagonal fuzzy number

We construct a pentagonal centroid by taking the centroid of all three planar figures into consideration and it is given by $G(\bar{x}_0, \bar{y}_0)$ as,

$$G(\bar{x}_0, \bar{y}_0) = \left(\frac{\frac{a_1+b_1+c_1}{3} + c_1 + \frac{c_1+d_1+e_1}{3}}{3}, \frac{\frac{1}{6}+\frac{1}{2}+\frac{1}{6}}{3} \right) \qquad = \left(\frac{a_1+a_2+5a_3+a_4+a_5}{9}, \frac{5}{18}\right)$$

$\therefore$ The centroid of a pentagonal fuzzy number is given by;

$$G(\bar{x}_0, \bar{y}_0) = \left(\frac{a_1+a_2+5a_3+a_4+a_5}{9}, \frac{5}{18}\right)$$

We note that the centroid of a non-normalized pentagonal fuzzy number is given by;

$$G(\bar{x}_0, \bar{y}_0) = \left(\frac{a_1+a_2+5a_3+a_4+a_5}{9}, \frac{5w_1}{18}\right)$$

2.6 Intuitionistic fuzzy sets

Let X be a universal set. An intuitionistic fuzzy set (IFS) $\tilde{A}$ in X is defined as $\tilde{A} = \{x, (\mu_{\tilde{A}}(x)\gamma_{\tilde{A}}(x)), x \in X\}$, where the functions $\mu_{\tilde{A}}(x)$, $\gamma_{\tilde{A}}(x)$. Severally, the degree of membership and degree of non-membership of the element $x \in X$ to the set $\tilde{A}$, that might be a subset of X, and for every $x \in X \le \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \le 1$. For each intuitionistic fuzzy set $\tilde{A} = \{x, (\mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)), x \in X)\}$ in X, $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \gamma_{\tilde{A}}(x)$ is called the hesitancy degree of x to lie $\tilde{A}$. If $\tilde{A}$ can be fuzzy set, then $\pi_{\tilde{A}}(x) = 0$ for all $x \in X$.

2.7 Intuitionistic fuzzy number

An IFS $\tilde{A} = \{x, (\mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)) : x \in X\}$ of the real line R is called an intuitionistic fuzzy number if

a) $\tilde{A}$ is convex for the membership function $\mu_{\tilde{A}}(x)$.

b) $\tilde{A}$ is concave for the non-membership function $\gamma_{\tilde{A}}(x)$.

c) $\tilde{A}$ is normal, that is there is some $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$, $\gamma_{\tilde{A}}(x_0) = 0$.

2.8 Generalized Intuitionistic pentagonal fuzzy number

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We tend to define intuitionistic fuzzy number $\tilde{A}$ as a generalized intuitionistic pentagonal fuzzy numbers (GIPFN) in the parameter.

$b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le a_4 \le b_4 \le a_5 \le b_5$

Denoted by

$\tilde{A} = \{(a_1, a_2, a_3, a_4, a_5)(b_1, b_2, b_3, b_4, b_5); w_{\tilde{A}}, v_{\tilde{A}}\}$, $0 \le w_{\tilde{A}}$, $v_{\tilde{A}} \le 1$, if its membership function and non-membership function are as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < a_1 \\ w_{\tilde{A}} - \dfrac{w_{\tilde{A}}(x-a_2)}{a_1 - a_2} & , \quad a_1 \le x \le a_2 \\ 1 + \dfrac{(w_{\tilde{A}}-1)(x-a_3)}{a_2 - a_3}, & \quad a_2 \le x \le a_3 \\ 1 + \dfrac{(w_{\tilde{A}}-1)(x-a_3)}{a_4 - a_3}, & \quad a_3 \le x \le a_4 \\ w_{\tilde{A}} - \dfrac{w_{\tilde{A}}(x-a_4)}{a_5 - a_4} & , \quad a_4 \le x \le a_5 \\ 0 & , \quad x < a_5 \end{cases}$$

$$\gamma_{\tilde{A}}(x) = \begin{cases} 1 & , \quad x < b_1 \\ 1 + \dfrac{(v_{\tilde{A}}-1)(x-b_1)}{b_2 - b_1} & , \quad b_1 \le x \le b_2 \\ v_{\tilde{A}} - \dfrac{v_{\tilde{A}}(x-b_2)}{b_3 - b_2} & , \quad b_2 \le x \le b_3 \\ \dfrac{v_{\tilde{A}}(x-a_3)}{b_4 - b_3} & , \quad b_3 \le x \le b_4 \\ v_{\tilde{A}} + \dfrac{(1-v_{\tilde{A}})(x-b_4)}{b_5 - b_4} & , \quad b_4 \le x \le b_5 \\ 1 & , \quad x < b_5 \end{cases}$$

2.9 Ranking of intuitionistic pentagonal fuzzy number

Let $\tilde{A}$ be an intuitionistic pentagonal fuzzy number. The centroid approach of $\tilde{A}$ is

$$R(\tilde{A}) = \frac{w_{\tilde{A}} S(\mu_{\tilde{A}}) + v_{\tilde{A}} S(\gamma_{\tilde{A}})}{w_{\tilde{A}} + v_{\tilde{A}}}$$

Here

$$S(\mu_{\tilde{A}}) = (x_0 \cdot y_0)$$
$$= \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}\right) \times \left(\frac{3w_{\tilde{A}} + 1}{9}\right)$$

$$S(\gamma_{\tilde{A}}) = (x_0 \cdot y_0)$$
$$= \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}\right) \times \left(\frac{3v_{\tilde{A}} + 5}{9}\right)$$

3 Single-Channel finite source imprecise queue models (FM/FM/1): (FCFS/n/M)

In certain cases, the units are sourced from a small pool of possible buyers. Once the unit is added to the queue, there will be one less unit to arrive, diminishing the chance of arrival. When a segment joins the pool of possible customers, it considered as served; the possibility of arrival is increased. The probability of arrival is determined by the quantity of potential customers who are able to enter the system.

As a result, if the entire consumer population is M, and n represents the number of consumers currently in the queuing system, the entry will come from an M-n number that is not yet in the queue. To determine the characteristics of this system, we must consider an equation for the probability of n customers in the system.

The expression P_n is given by P_0, $\tilde{\lambda}, \tilde{\mu}$ and M.

$$P_n = P_0\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \frac{M!}{(M-n)!}, \text{ Where}$$

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n\right]^{-1}$$

When there are n consumers in the system, the capacity of the system is $M - n$, which allows it to effectively manage more customers. As a consequence, the pace at which customers enter the system will be $\tilde{\lambda}(M-n)$. That is $C = 1$, that we have

$$\tilde{\lambda}_n = \begin{cases} \tilde{\lambda}(M-n) & , n = 0,1,2,...M \\ 0 & , n > M \end{cases}$$

$$\tilde{\mu}_n = \tilde{\mu}, n = 0,1,2,...M$$

An expression for P_0 and P_n are the same as for the model (FM/FM/C): (∞/FCFS)

(i) The probability that the system would be idle

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n\right]^{-1}$$

(ii) The probability that the system will have n customers

$$P_n = \frac{M!}{(M-n)!}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n P_0; n = 0,1,2,...M$$

(iii) The number of customers expected to be in the queue (or length)

$$\tilde{L}_q = \sum_{n=1}^{M}(n-1)P_n$$

$$= M - \left(\frac{\tilde{\lambda}+\tilde{\mu}}{\tilde{\lambda}}\right)(1-P_0)$$

(iv) The number of customers expected to be in the system

$$\tilde{L}_s = \sum_{n=0}^{\mu} nP_n = \tilde{L}_q + (1-P_0)$$

$$= M - \frac{\tilde{\mu}}{\tilde{\lambda}}(1-P_0)$$

(v) Expected queue waiting time

$$\tilde{W}_q = \frac{\tilde{L}_q}{\tilde{\mu}(1-P_0)} = \frac{1}{\tilde{\mu}}\left[\frac{M}{1-P_0} - \frac{\tilde{\lambda}+\tilde{\mu}}{\tilde{\lambda}}\right]$$

(vi) Expected system waiting time

$$\tilde{W}_s = \tilde{W}_q + \frac{1}{\tilde{\mu}} = \frac{1}{\tilde{\mu}}\left[\frac{M}{1-P_0} - \frac{\tilde{\lambda}+\tilde{\mu}}{\tilde{\lambda}} + 1\right]$$

3.1 Multiple Server imprecise queuing model with finite queue length - (FM / FM / S): (N / FCFS)

(i) The probability of ‘n’ customers in the system in the steady-State condition is:

$$P_n = \begin{cases} \dfrac{1}{n!}\left[\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right]^n P_0 \;;\; 0 < n < s \\ \dfrac{1}{s!\, s^{n-s}}\left[\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right]^n P_0 ;\; s \le n \le N \end{cases}$$

(ii) P_0 (i.e.) system shall be idle is:

$$P_0 = \left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n + \sum_{n=s}^{N}\frac{1}{s!s^{n-s}}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n\right]^{-1} \quad \text{... (1)}$$

(iii) The expected number of customers in the queue:

$$\tilde{L}_q = \sum_{n=s}^{N}(n-s)\, P_n \quad \text{... (2)}$$

$$= \frac{(s\rho)^s \rho}{s!\,(1-\rho)^2}\left[1-\rho^{N-s+1}-(1-\rho)(N-s+1)\,\rho^{N-S}\right]P_0$$

(iv) The expected number of customers in the system

$$\tilde{L}_s = \tilde{L}_q + \frac{\tilde{\lambda}\,(1\text{-}P_N)}{\tilde{\mu}} \quad \text{... (3)}$$

where $\lambda_{eff} = \tilde{\lambda}\,(1\text{-}P_N)$

$$= \tilde{L}_q + s - P_0 \sum_{n=0}^{s-1}\frac{(s-n)}{n!}\left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n$$

(v) The expected waiting time in the system

$$\tilde{W}_S = \frac{\tilde{L}_S}{\tilde{\lambda}\,(1\text{-}P_N)} \quad \text{... (4)}$$

(vi) The expected waiting time in the queue

$$\tilde{W}_q = \tilde{W}_S - \frac{1}{\tilde{\mu}}$$

$$= \frac{\tilde{L}_q}{\tilde{\lambda}\,(1\text{-}P_N)} \quad \text{... (5)}$$

4 Numerical precedents

I Single – Channel finite source imprecise queue models (FM/FM/1): (FCFS/n/M)

4.1 Numerical Precedent 1 : (For GPFN)

A service man repairs 5 machines. Consider a finite source queue model where both arrival and service rates are generalized pentagonal fuzzy numbers represented by $\tilde{\lambda} = (1,3,6,9,11;1)$ and $\tilde{\mu} = (20,22,25,28,30;1)$.

The membership function of the generalized pentagonal fuzzy number is defined by

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$$\mu_{\tilde{P}_{FN}} = \begin{cases} 0 & ; x \le 1 \\ \frac{1}{2}\left(\frac{x-1}{3-1}\right) & ; 1 \le x \le 3 \\ \frac{1}{2}+\left(1-\frac{1}{2}\right)\left(\frac{x-3}{6-3}\right) & ; 3 \le x \le 6 \\ 1 & ; x = 6 \\ \frac{1}{2}+\left(1-\frac{1}{2}\right)\left(\frac{9-x}{9-6}\right) & ; 6 \le x \le 9 \\ \frac{1}{2}\left(\frac{11-x}{11-9}\right) & ; 9 \le x \le 11 \\ 0 & ; x \ge 11 \end{cases}$$

Now we tend to calculate the ranking and applying the centroid of centroids technique

$$R(\tilde{\lambda}) = \left(\frac{a_1+a_2+5a_3+a_4+a_5}{9} \times \frac{5w_1}{18}\right)$$

$$R(\tilde{\lambda}) = 1.666667$$

Similarly $R(\tilde{\mu}) = 6.944444$

Let us contemplate c = 1, M = 5 machines (only one serviceman)

A queue arrival time $\tilde{\lambda} = (1,3,6,9,11;1)$ in hours and A queue service time $\tilde{\mu} = (20,22,25,28,30;1)$ in hours.

Here $\rho = \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)$

$\therefore \rho = 0.24$

(i) Probability that the system is idle

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}(\rho)^n\right]^{-1}$$

$$= [4.675123]^{-1}$$

$P_0 = 0.21390$

(ii) Probability of n customers

$$P_n = \frac{M!}{(M-n)!}(\rho)^n P_0,$$

$P_0 = 0.21390$

$P_1 = 0.25668$

$P_2 = 0.24641$

$P_3 = 0.17742$

$P_4 = 0.08516$

$P_5 = 0.02044$

(iii) Expected queue length

$$\tilde{L}_q = M - \left(\frac{\tilde{\lambda}+\tilde{\mu}}{\tilde{\lambda}}\right)(1-P_0)$$

$$= 5 - \left(\frac{1.67+6.94}{1.67}\right)(1-0.2139)$$

$$\tilde{L}_q = 0.947113$$

(iv) Expected customers in the system

$$\tilde{L}_s = \tilde{L}_q + (1-P_0)$$

$$= 0.947113 + (1-0.2139)$$

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$\tilde{L}_s$ = 1.733213

= Two customers in the system.

(v) Expected queue waiting time of a customers

$$\tilde{W}_q = \frac{\tilde{L}_q}{\tilde{\mu}(1-P_0)}$$

$$= \frac{0.947113}{6.94(1-0.2139)}$$

= 0.173606 hours = 10.4 minutes

(vi) Expected system waiting time

$$\tilde{W}_s = \tilde{W}_q + \frac{1}{\tilde{\mu}}$$

$$= 0.173606 + \frac{1}{6.94}$$

= 0.3177 hours

$\tilde{W}_s = 19$ minutes

4.2 Numerical precedent: 2 (For GIPFN)

Let the arrival rates at the same service rates as the intuitionistic pentagonal fuzzy number for $\tilde{\lambda}$ and $\tilde{\mu}$ and that can be represented as

$\tilde{\lambda} = \{(2,4,6,8,10)(1,3,6,9,11) : 0.5, 0.3\}$;

and $\tilde{\mu} = \{(21,23,25,27,29)(20,22,25,28,30) : 0.7, 0.2\}$.

The membership function and non-membership function of the intuitionistic pentagonal fuzzy number $\{(2,4,6,8,10)(1,3,6,9,11) : 0.5, 0.3\}$ is given as

$$\mu_{\tilde{\lambda}}(x) = \begin{cases} 0, & x < 2 \\ 0.5 - \dfrac{0.5(x-4)}{2-4}, & 2 \le x \le 4 \\ 1 + \dfrac{(0.5-1)(x-6)}{4-6}, & 4 \le x \le 6 \\ 1 + \dfrac{(0.5-1)(x-6)}{8-6}, & 6 \le x \le 8 \\ 0.5 - \dfrac{0.5(x-8)}{10-8}, & 8 \le x \le 10 \\ 0, & x < 10 \end{cases}$$

$$\gamma_{\tilde{\lambda}}(x) = \begin{cases} 1, & x < 1 \\ 1 + \dfrac{(0.3-1)(x-1)}{3-1}, & 1 \le x \le 3 \\ 0.3 - \dfrac{(0.3)(x-3)}{6-3}, & 3 \le x \le 6 \\ \dfrac{0.3(x-6)}{9-6}, & 6 \le x \le 9 \\ 0.3 + \dfrac{(1-0.3)(x-9)}{11-9}, & 9 \le x \le 11 \\ 1, & x > 11 \end{cases}$$

Similarly, the membership function and non-membership function of the remaining intuitionistic pentagonal fuzzy number arrivals will be written.

Now we tend to calculate the ranking and applying the centroids technique.

$R(\tilde{\lambda})$ = $R((2,4,6,8,10)\ (1,3,6,9,11) : 0.5, 0.3)$

= 2.51625

$R(\tilde{\mu}) = R((21,23,25,27,29)(20,22,25,28,30):0.7,0.2)$
$= 10.1543$

Here $\tilde{\rho} = \dfrac{\tilde{\lambda}}{\tilde{\mu}}$
$= 0.248$
$; \quad 0.25$

(i) Probability that the system is idle

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}(\rho)^n\right]^{-1}$$
$$= \{5.0234375\}^{-1}$$
$$P_0 = 0.1991$$

(ii) Probability of n customers

$$P_n = \frac{M!}{(M-n)!}(\rho)^n P_0 ,$$

$P_0 = 0.1991$
$P_1 = 0.248875$
$P_2 = 0.248875$
$P_3 = 0.186656$
$P_4 = 0.093328$
$P_5 = 0.02333$

(iii) Expected Queue length

$$\tilde{L}_q = M - \frac{(\tilde{\lambda}+\tilde{\mu})}{\tilde{\lambda}}(1-P_0)$$
$$= 5 - \left(\frac{2.51625+10.1543}{2.51625}\right)(1-0.1991)$$
$$= 0.967077$$

(iv) Expected customers in the system

$$\tilde{L}_s = \tilde{L}_q + (1-P_0)$$
$= 0.967077 + (1 – 0.1991)$
$= 1.76798$
= 2 customers in the system

(v) Expected queue waiting time of a customers

$$\tilde{W}_q = \frac{\tilde{L}_q}{\tilde{\mu}(1-P_0)}$$
$$= \frac{0.967077}{10.1543(1-0.1991)}$$
$= 0.118914$ hours
$= 7.1348$ mins ; 7 mins

(vi) Expected system waiting time

$$\tilde{W}_s = \tilde{W}_q + \frac{1}{\tilde{\mu}}$$
$= 0.217394$ hours
$= 13.04$ mins
; 13 mins

4.3 Numerical precedent : 3

II Multiple Server imprecise queuing model with finite queue length - (FM / FM / S): (N / FCFS)

(a) For GPFN

Let us contemplate S = 3, N = 5 and we consider both arrival rate and service rates are generalized pentagonal fuzzy numbers represented by $\tilde{\lambda} = (1,3,6,9,11;\ 1)$ and $\tilde{\mu} = (20, 22, 25, 28, 30;\ 1)$

Here also we tend to calculate the ranking and applying the centroid of centroids technique. We get $R(\tilde{\lambda}) = 1.667$, $R(\tilde{\mu}) = 6.944$ and $\tilde{\rho}=0.24$

(i) Probability that the system is idle

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n + \sum_{n=s}^{N} \frac{1}{s!\, s^{n-s}} \left[\frac{\tilde{\lambda}}{\tilde{\mu}}\right]^n \right]^{-1}$$

$$= \left[\sum_{n=0}^{2} \frac{1}{n!}(0.24)^n + \sum_{n=3}^{5} \frac{1}{s!\, s^{n-s}} \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \right]^{-1}$$

$$= \left[\frac{1}{0!}(0.24)^0 + \frac{1}{1!}(0.24)^1 + \frac{1}{2!}(0.24)^2 + \frac{1}{3!3^{3-3}}(0.24)^3 + \frac{1}{3!3^{4-3}}(0.24)^4 + \frac{1}{3!3^{5-3}}(0.24)^5 \right]^{-1}$$

$= [1.2713]^{-1}$

$P_0 = 0.7866$

(ii) The Expected number of customers in the queue

$$\tilde{L}_q = \sum_{n=s}^{N} (\text{n-s})\, P_n$$

$$\tilde{L}_q = \sum_{n=3}^{5} (\text{n-s})\, P_n$$

$$= [0P_3 + 1.P_4 + 2P_5]$$

$$= \left[\frac{1}{3!3^{4-3}}(0.24)^4 + \frac{2}{3!3^{5-3}}(0.24)^5\right] P_0$$

$$\therefore \tilde{L}_q = 0.000168$$

(iii) The Expected number of customers in the system

$$\tilde{L}_s = \tilde{L}_q + \frac{\tilde{\lambda}\,(1 - P_N)}{\tilde{\mu}}$$

$= 0.000168 + (0.24)\,(1 - P_5)$

where $1 - P_5 = 1 - \left[\frac{1}{3!3^2}(0.24)^5(0.7866)\right]$

$= 1 - 0.0000116$

$= 0.99998$

$\therefore \tilde{L}_s = 0.24017$

(iv) Expected waiting time in the system

$$\tilde{W}_s = \frac{\tilde{L}_s}{\tilde{\lambda}\,(1 - P_N)} = \frac{0.24017}{1.66667}$$

$$\tilde{W}_S = 0.14410 = 8.646 \text{ mins.}$$

(v) Expected waiting time in the queue

$$\tilde{W}_q = \tilde{W}_S - \frac{1}{\tilde{\mu}}$$

$$= 0.14410 - \frac{1}{6.94}$$

$$\tilde{W}_q = 0.0000078 = 0.0005 \text{ mins.}$$

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4.4 Numerical Precedent : 4

(b) For GIPFN

Let us consider $S = 3$, $N = 5$ and Let us take the arrival rate and service rates are intuitionistic pentagonal fuzzy numbers represented by
$\tilde{\lambda} = \{(2, 4, 6, 8, 10)\ (1, 3, 6, 9, 11) : 0.5, 0.3\}$ and
$\tilde{\mu} = \{(21, 23, 25, 27, 29)\ (20, 22, 25, 28, 30) : 0.7, 0.2\}$ Now we tend to calculate the ranking and applying the centroids technique. We get
$R(\tilde{\lambda}) = 2.52$, $R(\tilde{\mu}) = 10.154$ and $\tilde{\rho} = 0.25$
Now substituting the above values in equ (1), (2), (3), (4) and (5) We get
$P_0 = 1.2841$, $\tilde{L}_q = 0.000033$ $\tilde{L}_S = 0.250032$, $\tilde{W}_S = 0.099207 = 5.95$ mins. and
$\tilde{W}_q = 0.00068 = 0.04$ mins respectively.

Table 1. Analogy between GPFN and GIPFN Results

S. No.	Membership functions	$\tilde{L}_q$	$\tilde{L}_s$	$\tilde{W}_q$	$\tilde{W}_s$
	For Single Server				
1.	Generalized pentagonal fuzzy number	0.947113	1.733213	0.174 hours (or) 10.4 mins	0.32 hours (or)19 mins
2.	Generalized Intuitionistic pentagonal fuzzy numbers	0.967077	1.767980	0.12 hours (or) 7 mins	0.2 hours (or)13mins
	For Multi Server				
1.	Generalized pentagonal fuzzy number	0.000168	0.24017	0.000008 hours (or) 0.0005 mins	0.144 hours (or) 8.6 mins
2.	Generalized Intuitionistic pentagonal fuzzy numbers	0.000033	0.250032	0.00068 hours (or) 0.04 mins	0.099207 hours (or) 5.95 mins

Finally, for single server, GIPFN has greater queue length and customers than GPFN both in queue and in system. And also we obtained GIPFN has lesser waiting time than GPFN both in queue and in system. For multi-server, GIPFN has lesser expected number of customers in queue and waiting time in system and also we got GIPFN has greater expected number of customers in system and waiting time in queue than GPFN (From Table 1). As a result, we can encapsulate impreciseness or vagueness in any decision-making system using the generalized pentagonal fuzzy number.

Final thoughts

The performance of pentagonal fuzzy numbers in a finite source queue model for first-come, first served bases on a single-server and multi server system is examined in this article. When it comes to transforming fuzzy numbers to crisp values, centroid of centroids method is particularly effective. The features of an improved from of non-normalized (generalized) pentagonal fuzzy numbers (GPFN) and intuitionistic pentagonal fuzzy numbers (GIPFN) are presented in this study. The non-normalized form of the centroid of a pentagonal fuzzy number has been developed and expanded. The fields of arrival and service times are hazy. In the same way, the finite source queue system has a hazy aspect. The performance of the finite source queuing system is demonstrated numerically.

References

Atanassov, K.T., "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, *Elsevier Science Publications,* Vol. 20, No. 1, pp. 87-96, (1986).

Banerjee, S., and Roy, T.K., "Arithmetic Operations on Generalized Trapezoidal Fuzzy Number and its Applications", *Turkish Journal of Fuzzy Systems*, Vol. 3, No. 1, pp. 16-44, ISSN: 1309-1190, (2012).

Bansal, A., "Trapezoidal Fuzzy numbers (a,b,c,d): Arithmetic Behavior", *International Journal of Physical and Mathematical Sciences*, pp. 39-44, ISSN: 2010-1791, (2011).

Bekheet, S., "Polygon Fuzzy Numbers", *Africa Development and Resources Research Institute Journal*, Vol. 3, No. 3, pp. 52-63, (2013).

Black, M., "Reasoning with Loose Concepts", *Dialogue*, Vol. 2, pp. 1-12, (1963).

Dijkman, J.G., Haeringen, H. van., and Lange, S.J. de., "Fuzzy Numbers", *Journal of Mathematical Analysis and Applications*, Vol. 92, No. 2, pp. 301-341, (1983).

Dubois, D., and Prade, H., "Operations on Fuzzy Numbers", *International Journal of System Sciences,* Vol. 9, pp. 613-626, (1978).

Gani, A.N., and Assarudeen, A.N.M., "A New Operation on Triangular Fuzzy number for Solving Fuzzy Linear Programming Problem", *Applied Mathematical Sciences,* Vol. 6, No. 11, pp. 525-532, (2012).

Hanss, M., "Applied Fuzzy Arithmetic: An Introduction with Engineering Applications", Springer - Verlag Berlin Heidelberg, ISBN 978-81-8489-300-7, (2005).

Helen, R., and Uma, G., "A New Operation and Ranking on Pentagon Fuzzy Number", *International Journal of Mathematical Sciences and Applications,* Vol. 5, No. 2, pp. 341-346, (2015).

Hellendoorn, H., "Fuzzy Numbers and Approximate Reasoning", Annales Univ. Sci. Budapest., *Sect. Comp,* Vol. 12, pp. 113-119, (1991).

Jain, R., "A Procedure for Multiple-aspect Decision using Fuzzy Sets", *International Journal of System Sciences,* Vol. 8, pp. 1-7, (1977).

Jain, R., "Tolerance Analysis Using Fuzzy Sets", *International Journal of System.*

Pathinathan. T., Ponnivalavan. K. and Mike Dison. E. "Different Types of Fuzzy Numbers and Certain Properties", Journal of Computer and Mathematical Sciences, Vol.6 (11), 631-651, November 2015 ISSN 0976-5727 (Print) ISSN 2319-8133 (Online).

R. Srinivasan, T. Nakkeeran, K. Renganathan, V. Vijayan., "The performance of pentagonal fuzzy numbers in finite source queue models using Pascal's triangular graded mean", Elsevier Ltd,(2020).

Shawky (2000): The machine interference model, M/M/C/K/N with balking, reneging and spares, Opsearch, Vol. 37, pp. 25-35.

Saima Jabee, Vandana Malviya, Performance Measures of M/M/C/N Queuing Model, IJESRR, Volume-2, Issue-1, ISSN 2348-6457.