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A Precise Study on Analyzation of FM/FD/1 imprecise queuing model under various fuzzy numbers: Execution Proportions by Pascal's triangular graded mean technique

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ABSTRACT

We propose an FM/FD/1 imprecise queuing model with an indefinite limit under various fuzzy numbers, Arrival (landing) and service (administration) rates are assumed to be different types of fuzzy numbers. To convert fuzzy arrival and service times into a crisp value of using Pascal's triangular graded mean for various fuzzy numbers. By applying this technique, here we determine the execution proportions. Using the typical queuing hypothesis, the major goal is to evacuate ambiguity before the exhibiting parameters are processed. The numerical examples are shown to demonstrate the methodology's validity in reality. A comparative illustration corresponding to each fuzzy number is accomplished. Finally, the miniature variables were exposed to sensitivity analysis.

Keywords: Execution proportions, Fuzzy numbers, Pascal's triangular graded mean technique, FM/FD/1 queuing model.

1 Introduction

A queue is made up of at least one queue or one or more remodeled offices that are all organized according to a set of rules. In order to pursue dissemination in queuing hypothesis, the fuzzy parameters landing rate $\left(\tilde{\lambda}\right)$ and administration rate $\left(\tilde{\mu}\right)$ are necessary. The reason for this is its ability to model difficulties like ambiguity and imprecision quantitatively and subjectively. Queuing models are widely used in administrative organizations.

One of these application zones is real-life scenarios, which are managed through a single channel. We explored a variety of lining models, ranging from those with consistent fresh properties to those with fuzzy qualities. In this approach, Zadeh depicted certain standard models as having articulations as a possibility. In a variety of real-world scenarios, fuzzy queuing models are more practical than classical queuing models.

1.1 Literature Review

Zadeh, L.A. [16] introduced fuzzy sets in 1965. Kanufmann, A., [3] suggested an introduction to the theory of fuzzy subsets in 1975. Zadeh, L.A., [15] developed fuzzy sets as a basis for a theory of possibility in 1978. R.B. Cooper,[1] proposed an introduction to queuing theory in 1981.H.J. Zimmermann,[17] examined fuzzy set theory and mathematical programming, and Yager, R. R.,[14] discussed a characterization of the extension principle in 1986. Li, R. J and Lee, E. S. [5] investigated analysis of fuzzy queues in 1989.

Negi, D. S. and Lee, E. S.,[6] illustrated analysis and simulation of fuzzy queues in 1992. Lofti, A. Z. [4] explored fuzzy sets, fuzzy logic and fuzzy systems in 1996. Chen, S. P., [2] scrutinized a mathematics programming approach to the machine interference problem with fuzzy parameters in 2006. Novak, and Watson, R.,[7] were studied determining an adequate probe separation for estimating the arrival rate in an M/D/1 queue using single–packet probing in 2009. N. Subramaniyam,[8] demonstrated probability and queuing theory and Timothy, J.R.[10] suggested a fuzzy logic that might be used in engineering applications in 2010. Shortle, J. F., Thompson, J. M., Gross, D. and Harris, C. M.[9] explained fundamentals of queuing theory in 2018.

Usha Prameela, K. and Pavan Kumar, ([11],[12])derived execution proportions of multi server queuing model with pentagonal fuzzy number: DSW algorithm approach and investigated $FM/FE_k/1$ queuing model with Erlang service under various types of fuzzy numbers in 2019.Finally,Usha Prameela, K. and Pavan Kumar,[13] developed and analyzed perceptionization of FM/FD/1 queuing model under various fuzzy numbers in 2020.

Both the characteristics, that is, the inter-landing periods and administration times, are essential to pursue particular appropriations in the typical lining model. In typical practice, the landing rate and administration rate are frequently represented by etymological phrases, such as high, low, extremely low, and moderate, which are best represented by fuzzy sets.

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The simplest queue with deterministic service time is the FM/FD/1 queue, which has a number of applications in production management, telecommunications networks, and other domains. For placing fuzzy numbers, various approaches have been developed. In the present research, the Pascal's triangular graded mean technique for various fuzzy numbers is to create appropriate maps to convert fuzzy numbers into real (crisp) numbers, a process known as defuzzification.
The design of this paper pursues: Segment 1 provides an overview, segment 2 contains some basic definitions, segment 3 explains some presumptions and notations, segment 4 describes proposed fuzzy queuing model. Segment 5 depicts Pascal's triangular graded mean technique for various fuzzy numbers, segment 6 presents numerical precedents, segment 7 provides results and discussions, segment 8 examines sensitivity analysis and concludes the paper.

2 Basic definitions

2.1 Fuzzy Set

If x belongs to the classical set $\tilde{A}$ in the pair $\left(x, \mu_{\tilde{A}}(x)\right)$, and the second set $\mu_{\tilde{A}}(x)$ belongs to the interval [0,1], then set $\tilde{A}$ is called a fuzzy set. A membership function $\mu_{\tilde{A}}(x)$ is defined here.

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) : x \in \tilde{A}, \mu_{\tilde{A}}(x) \in [0,1]\right\}$$

2.2 Fuzzy number

A fuzzy set $\tilde{A}$ in $\tilde{R} \to [0,1]$ with a membership function is called a fuzzy number,

1) $\tilde{A}$ is normal
2) $\tilde{A}$ is fuzzy concave.
3) $\mu_{\tilde{A}}$ is upper semi continuous.
4) Supp $\tilde{A}$ is bounded.

3 Presumptions and notations

In this paradigm, accompanying presumptions are given as:

i) With only one server, the FM/FD/1/∞/FCFS queuing paradigm has no bounds.
ii) Arrival times that are exponentially publicized.
iii) Deterministic service appropriation, (i.e.) fixed service appropriation.
iv) The landing rate and administration rate are both ambiguous numbers.

Instances of notations

$\tilde{\mu}$ = The average number of customers refurbished per unit of time
$\tilde{\lambda}$ = average number of customers arriving per unit of time.
$\tilde{L}_s$ = average number of customers in the system.
$\tilde{L}_q$ = average number of customers holding up in the queue.
$\tilde{W}_s$ = average sustaining time of a customer in the framework.
$\tilde{W}_q$ = average upholding time of a customer in the queue.

4 Proposed fuzzy queuing model

We envisage a single-server queuing paradigm with a first-come, first-served (FCFS) policy. The notation (FM/FD/1): (∞/FCFS) is indicated in Kendall's notation. FD stands for fuzzified steady (Constant) dispersion with administration rate $\tilde{\mu}$, while FM stands for fuzzified exponential dispersion with landing rate $\tilde{\lambda}$. This is a stochastic process whose state space is the set $\{0,1,2,3,....\}$ where the value represents the number of customers in the system, encompassing any entity at present in administration. There is no limit to the amount of customers it can hold because it is infinitely large. The execution proportions of the proposed model are given as below.

(i) The expected number of consumers within the system is:

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$$\tilde{L}_s = \tilde{\rho} + \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})}, \left(\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}}\right) \quad \text{... (1)}$$

(ii) The expected number of consumers within the queue is:

$$\tilde{L}_q = \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})} \quad \text{... (2)}$$

(iii) The expected waiting time of a client spends within the queue is:

$$\tilde{W}_q = \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})\tilde{\mu}} \quad \text{... (3)}$$

(iv) The expected holding up time of a client spends within the system is:

$$\tilde{W}_s = \frac{1}{\tilde{\mu}} + \frac{\tilde{\rho}^2}{2(1-\tilde{\rho})\tilde{\mu}} \quad \text{... (4)}$$

5 Pascal's triangular graded mean technique

In this section, the method for converting fuzzy numbers into crisp numbers is explained. Eight sorts of fuzzy numbers: triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal and decagonal fuzzy numbers are executed with the Pascal's triangular graded mean technique. We evaluate the following scenarios to begin attributing the innovation for this system:
The process of converting fuzzy numbers into crisp numbers is described in this section.

5.1 Triangular fuzzy number (case 1)

Let $\tilde{A} = \{a_1, a_2, a_3\}$ be a triangular fuzzy number, we can take the coefficient of fuzzy numbers from the triangles of Pascal's and use simple approach of probability, we get the following formula,

$$\mathrm{P}(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4} \quad \text{... (5)}$$

The coefficients a_1, a_2, a_3 are 1, 2 and 1 respectively. This procedure is simply taken from the triangles of Pascal's. These are useful to take into consideration the coefficients of fuzzy variables are Pascal triangular numbers, and we just add and divided by the total of Pascal numbers.

5.2 Trapezoidal fuzzy number (case 2)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4\}$ be a trapezoidal fuzzy number, we can take the coefficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 3a_2 + 3a_3 + a_4}{8} \quad \text{... (6)}$$

5.3 Pentagonal fuzzy number (case 3)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5\}$ be a pentagonal fuzzy number, we can take the coefficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4 + a_5}{16} \quad \text{... (7)}$$

5.4 Hexagonal fuzzy number (case 4)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ be a Hexagonal fuzzy number, we can take the coefficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 5a_2 + 10a_3 + 10a_4 + 5a_5 + a_6}{32} \quad \text{... (8)}$$

5.5 Heptagonal fuzzy number (case 5)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a heptagonal fuzzy number, we can take the co-efficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 6a_2 + 15a_3 + 20a_4 + 15a_5 + 6a_6 + a_7}{64} \quad \text{... (9)}$$

5.6 Octagonal fuzzy number (case 6)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ be an octagonal fuzzy numbers, we can take the co-efficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 7a_2 + 21a_3 + 35a_4 + 35a_5 + 21a_6 + 7a_7 + a_8}{128} \quad \text{... (10)}$$

5.7 Nonagonal fuzzy number (case 7)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ be a Nonagonal fuzzy number, we can take the co-efficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 8a_2 + 28a_3 + 56a_4 + 70a_5 + 56a_6 + 28a_7 + 8a_8 + a_9}{256} \quad \text{... (11)}$$

5.8 Decagonal fuzzy number (case 8)

Let $\tilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$ be a Decagonal fuzzy number, we can take the co-efficient of fuzzy numbers from the triangles of Pascal's and use the simple approach of probability, we get the following formula:

$$P(\tilde{A}) = \frac{a_1 + 9a_2 + 36a_3 + 84a_4 + 126a_5 + 126a_6 + 84a_7 + 36a_8 + 9a_9 + a_{10}}{512} \quad \text{... (12)}$$

6 Numerical Precedents

We describe the numerical precedents that go along with them by envisioning various fuzzy numbers. Numerical examples are presented for each case. Consider an FM/FD/1/∞/FIFO model where both the entry rate and overhauled rate are hazy. The administration must obtain the mean of line as well as system length and average waiting time of line and system for this fuzzy queuing model.

Precedent 1

Let us contemplate that both landing rate and administration rates are triangular fuzzy numbers in a FM/FD/1/∞/FIFO queuing model represented by $\tilde{\lambda} = [3, 4, 5]$ and $\tilde{\mu} = [13, 14, 15]$.

Applying the graded mean integration representation for triangular fuzzy numbers, according to the above equ (5), we acquire $\tilde{\lambda} = 4$, and similarly we obtain $\tilde{\mu} = 14$.

Here $\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} = \frac{4}{14} = 0.2857$.

From equations (1), (2), (3) and (4), we obtain the performance measures for triangular fuzzy numbers are:

$\tilde{L}_s = 0.3428$, $\tilde{L}_q = 0.0571$, $\tilde{W}_q = 0.0041$ and $\tilde{W}_s = 0.0755$.

Precedent 2

Concede that both entry rate and overhauled rates are trapezoidal fuzzy numbers represented by $\tilde{\lambda} = [3, 4, 5, 6]$ and $\tilde{\mu} = [13, 14, 15, 16]$. Applying the Pascal's graded mean for trapezoidal fuzzy numbers. According to the above equation (6), we obtain $\tilde{\lambda} = 4.5$ and $\tilde{\mu} = 14.5$, $\therefore \tilde{\rho} = 0.3103$.

From equations (1), (2), (3) and (4), we obtain, the performance measures for trapezoidal fuzzy numbers are:

$\tilde{L}_s = 0.3802$, $\tilde{L}_q = 0.0698$, $\tilde{W}_q = 0.0048$ and $\tilde{W}_s = 0.0738$

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Precedent 3

Confess that both arrival rate and service rates are pentagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7)$ and $\tilde{\mu} = (13,14,15,16,17)$. According to the above equation (7), we get $\tilde{\lambda} = 5$ and $\tilde{\mu} = 15$.
Therefore, $\tilde{\rho} = 0.3333$, from equations (1), (2), (3) &(4), we obtain, the performance measures for pentagonal fuzzy numbers are:

$\tilde{L}_s = 0.4167$, $\tilde{L}_q = 0.0833$, $\tilde{W}_q = 0.0056$ and $\tilde{W}_s = 0.0722$

Precedent 4

Acknowledge that both the landing rate and overhauled rates are hexagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7,8)$ and $\tilde{\mu} = (13,14,15,16,17,18)$.
Pursuant to the above equation (8), we obtain

$\tilde{\lambda} = 5.5$ and $\tilde{\mu} = 15.5$, $\therefore \tilde{\rho} = 0.3548$

From equations (1), (2), (3) & (4), we obtain the performance measures for hexagonal fuzzy numbers are:

$\tilde{L}_s = 0.4524$, $\tilde{L}_q = 0.0976$, $\tilde{W}_q = 0.0063$ and $\tilde{W}_s = 0.0708$.

Precedent 5

Recognize that both the entry rate and administration rates are heptagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7,8,9)$ and $\tilde{\mu} = (13,14,15,16,17,18,19)$. According to the above equation (9), we obtain $\tilde{\lambda} = 6$ and $\tilde{\mu} = 16$. Hence $\tilde{\rho} = 0.375$.

From equations (1), (2), (3) and (4), we obtain the performance measures for heptagonal fuzzy numbers are:

$\tilde{L}_s = 0.4875$, $\tilde{L}_q = 0.1125$, $\tilde{W}_q = 0.0070$ and $\tilde{W}_s = 0.0695$.

Precedent 6

Let us consider both the advent rate and service rates are octagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7,8,9,10)$ and $\tilde{\mu} = (13,14,15,16,17,18,19,20)$. Pursuant to the above equation (10), we obtain $\tilde{\lambda} = 6.5$ and $\tilde{\mu} = 16.5$, $\therefore \tilde{\rho} = 0.394$. Substituting $\tilde{\rho}$ values in equations (1), (2), (3) and (4), we obtain $\tilde{L}_s = 0.5221$, $\tilde{L}_q = 0.1281$, $\tilde{W}_q = 0.0078$ and $\tilde{W}_s = 0.0684$.

Precedent 7

Assimilate that both the arrival rate and service rates are nonagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7,8,9,10,11)$ and $\tilde{\mu} = (13,14,15,16,17,18,19,20,21)$. Applying to the above equation (11), we obtain $\tilde{\lambda} = 7$ and $\tilde{\mu} = 17$. Therefore, $\tilde{\rho} = 0.412$. From equations (1), (2), (3) and (4), we obtain $\tilde{L}_s = 0.5563$, $\tilde{L}_q = 0.1443$, $\tilde{W}_q = 0.0085$ and $\tilde{W}_s = 0.0673$.

Precedent 8

Acquiesce in both the influx rate and service rates are decagonal fuzzy numbers represented by $\tilde{\lambda} = (3,4,5,6,7,8,9,10,11,12)$ and $\tilde{\mu} = (13,14,15,16,17,18,19,20,21,22)$. According to the above equation (12), we obtain $\tilde{\lambda} = 7.5$ and $\tilde{\mu} = 17.5$. Hence $\tilde{\rho} = 0.4286$.

From equations (1), (2), (3) and (4) we obtain the execution proportions of the proposed model are $\tilde{L}_s = 0.5893$, $\tilde{L}_q = 0.1607$, $\tilde{W}_q = 0.0092$ and $\tilde{W}_s = 0.0663$.

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7 Results and Discussions

The obtained execution measures are shown in Table 1, which elucidate various estimations for a broad range of membership functions considered as:

Table 1: Performance measures of $\tilde{L}_q$, $\tilde{L}_s$, $\tilde{W}_q$ and $\tilde{W}_s$

Membership functions	$\tilde{L}_q$	$\tilde{L}_s$	$\tilde{W}_q$	$\tilde{W}_s$
Triangular fuzzy number	0.0571	0.3428	0.0041	0.0755
Trapezoidal fuzzy number	0.0698	0.3802	0.0048	0.0738
Pentagonal fuzzy number	0.0833	0.4167	0.0056	0.0722
Hexagonal fuzzy number	0.0976	0.4524	0.0063	0.0708
Heptagonal fuzzy number	0.1125	0.4875	0.0070	0.0695
Octagonal fuzzy number	0.1281	0.5221	0.0078	0.0684
Nonagonal fuzzy number	0.1443	0.5563	0.0085	0.0673
Decagonal fuzzy number	0.1607	0.5893	0.0092	0.0663

The graphical representations of Table 1 are demonstrated in Figure 1.

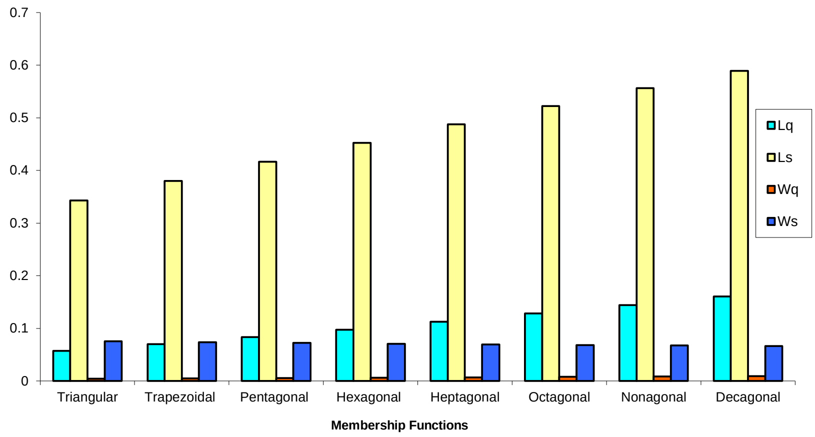

Figure 1. Graphical representation of Table 1

The positioning technique produces different arrangements of real variables, such as landing and administration rates for various types of fuzzy numbers as shown in Table 1 and its graphical representation in Figure 1.

8 Sensitivity Analysis

A sensitive analysis is performed with various fuzzy numbers (triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal and decagonal fuzzy numbers) based on predictions in this segment to examine sensitivity of the model. The sensitivity of this model is analysed by varying i.e., decreasing or increasing the values of anyone parameter $(\tilde{\lambda}, \tilde{\mu})$ while leaving other parameter unchanged. Now we tend to calculate the ranking and applying the Pascal's graded mean technique in each cases; For example in case of triangular fuzzy number when the values of the parameter $\tilde{\lambda}$ is decreased by 0.5 (i.e.) when $\tilde{\lambda} = [2.5, 3.5, 4.5]$ keeping $\tilde{\mu}$ as it is, then the performance measures $\tilde{L}_q$, $\tilde{L}_s$, $\tilde{W}_q$, $\tilde{W}_s$ of Table 2 are less than or equal to the performance measures of Table 1, when $\tilde{\lambda}$ is increased by 0.5, i.e., when $\tilde{\lambda} = [3.5, 4.5, 5.5]$. Keeping $\tilde{\mu}$ as it is, then the performance measures of Table 2 are greater than or equal to the performance measures of Table 1.

In case of trapezoidal when $\tilde{\mu}$ is decreased by 0.5 and the performance measures of this model are greater than or equal to the values of Table 1 if $\tilde{\mu}$ is increased by 0.5, then the performance measures are less than or equal to the values of Table 1 and by proceeding in this manner, finally in case of decagonal when $\tilde{\mu}$ is decreased and increased by 0.5 then also the results are almost the same. We also discovered that using additional types of fuzzy numbers encourages us to gather more accurate data and make more flexible decisions in the framework.

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Table 2: Sensitivity analysis of execution proportions

	Membership functions	$\tilde{L}_q$	$\tilde{L}_s$	$\tilde{W}_q$	$\tilde{W}_s$
1.	Triangular fuzzy number				
i)	decrease $\tilde{\lambda}$ values by 0.5	0.0417	0.2917	0.0030	0.0744
ii)	increase $\tilde{\lambda}$ values by 0.5	0.0753	0.3953	0.0054	0.0768
2.	Trapezoidal fuzzy number				
i)	decrease $\tilde{\mu}$ values by 0.5	0.0753	0.3953	0.0054	0.0768
ii)	increase $\tilde{\mu}$ values by 0.5	0.0643	0.3643	0.0043	0.0710
3.	Pentagonal fuzzy number				
i)	decrease $\tilde{\lambda}$ values by 0.5	0.1062	0.4729	0.0071	0.0737
ii)	increase $\tilde{\lambda}$ values by 0.5	0.0643	0.3643	0.0043	0.0710
4.	Hexagonal fuzzy number				
i)	decrease $\tilde{\mu}$ values by 0.5	0.1062	0.4729	0.0071	0.0737
ii)	increase $\tilde{\mu}$ values by 0.5	0.0876	0.4276	0.0055	0.0680
5.	Heptagonal fuzzy number				
i)	decrease $\tilde{\lambda}$ values by 0.5	0.0876	0.4276	0.0055	0.0680
ii)	increase $\tilde{\lambda}$ values by 0.5	0.1425	0.5525	0.0089	0.0714
6.	Octagonal fuzzy number				
i)	decrease $\tilde{\mu}$ values by 0.5	0.1425	0.5525	0.0089	0.0714
ii)	increase $\tilde{\mu}$ values by 0.5	0.1165	0.4965	0.0069	0.0657
7.	Nonagonal fuzzy number				
i)	decrease $\tilde{\lambda}$ values by 0.5	0.1165	0.4965	0.0069	0.0657
ii)	increase $\tilde{\lambda}$ values by 0.5	0.1729	0.6129	0.0102	0.0690
8.	Decagonal fuzzy number				
i)	decrease $\tilde{\mu}$ values by 0.5	0.1729	0.6129	0.0102	0.0690
ii)	increase $\tilde{\mu}$ values by 0.5	0.1521	0.5721	0.0085	0.0640

The graphical representations of Table 2 are demonstrated in Figure 2.

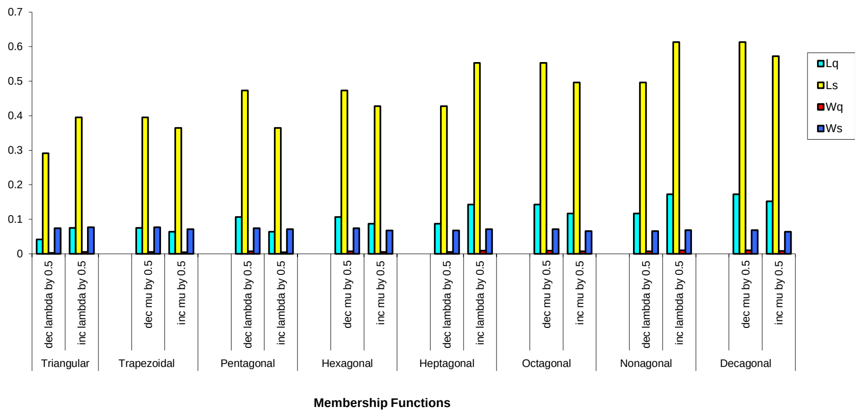

Figure 2. Graphical representation of Table 2

8.1 Limitations

The arrival rate is not stationary, which is an evident restriction. It is state dependent. The steady state solution is provided by the queuing model. Unlike the conventional model, which assumes that arrivals follow a Poisson process with exponentially distributed service times, in many real-world circumstances, the arrival rate is more possibility than probabilistic.

A single channel model is the only option. The average rate of arrival is lower than the average rate of service, i.e., $\tilde{\lambda} < \tilde{\mu}$.

Conclusion

We presume that the fuzzy set hypothesis has been more closely linked to the lining hypothesis during the inter-landing and administration times. Entity length, line length, entity time, line time, and other execution proportions are similarly ambiguous. Pascal's graded mean technique is extremely effective in converting fuzzy numbers to crisp values. Time of arrival and service are hazy fields. In this paper, sensitivity analysis is also performed to provide more than one solution of values in the queuing system with different types of membership functions. The proficiency of Pascal's graded mean estimation is demonstrated by numerical precedents.

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