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SOME PROPERTIES ON $(\mathfrak{t}_1,\mathfrak{t}_2)$ – INTUITIONISTIC MULTI FUZZY SUBRING OF A RING

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ABSTRACT

In this paper, we have characterized $(\mathfrak{t}_1,\mathfrak{t}_2)$- Intuitionistic Multi fuzzy subring of a ring R and talked about a portion of its properties by utilizing (α,β)- cuts. Additionally, we have characterized $(\mathfrak{t}_1,\mathfrak{t}_2)$- intuitionistic multi fuzzy cosets of a ring and demonstrated a few related theorems with examples.

Keywords:

$(\mathfrak{t}_1,\mathfrak{t}_2)$-Intuitionistic Fuzzy Set ($(\mathfrak{t}_1,\mathfrak{t}_2)$-IFS), $(\mathfrak{t}_1,\mathfrak{t}_2)$-Intuitionistic Multi Fuzzy Set ($(\mathfrak{t}_1,\mathfrak{t}_2)$-IMFS), $(\mathfrak{t}_1,\mathfrak{t}_2)$-Intuitionistic Multi Fuzzy Subring ($(\mathfrak{t}_1,\mathfrak{t}_2)$-IMFSR) , $(\mathfrak{t}_1,\mathfrak{t}_2)$-Intuitionistic Multi Fuzzy Normal Subring(IMFNSR), (α,β)- cuts, Homomorphism(_homo).

1. Introduction

The fuzzy set speculation introduced by L. A Zadeh [19] has shown huge application in many fields of study. The chance of a fuzzy set is welcome since it handles weakness and lack of definition that ordinary sets couldn't address. In fuzzy set theory membership function of an element in a single value between 0 and 1, therefore, a generalization of the fuzzy set was introduced by Atanassov[1] called intuitionistic fuzzy set which deals with the degree of non-membership function and the degree of membership. Following quite a while S.Sabu [14] presented the hypothesis of multi fuzzy sets as far as multi-dimensional membership function. The idea of fuzzy subgroups was presented by Rosenfeld [13]. Biswas [5] applied the idea of intuitionistic fuzzy sets to the hypothesis of groups and considered intuitionistic fuzzy subgroups of a group.Marashdeh and Salleh [20] presented the idea of intuitionistic fuzzy rings in view of the thought of fuzzy space. The idea of ŧ-intuitionistic fuzzy subgroups and ŧ-intuitionistic quotient groups has proactively been presented by P.K. Sharma [18,24]. Likewise, he [17] presented the idea of the ŧ-intuitionistic fuzzy subring of a ring. P.Dheena and B.Anitha [21] presented the possibility of $(\mathfrak{t}_1,\mathfrak{t}_2)$- Intuitionistic fuzzy set and talked about a portion of its properties. In this paper, we introduced the possibility of $(\mathfrak{t}_1,\mathfrak{t}_2)$ intuitionistic multi fuzzy subring and their properties.

2. Preliminaries

Definition 2.1[19] Let $X \neq \emptyset$. A fuzzy subset A of x is characterized by a function $A: x \to [0,1]$

Definition 2.2 [1] An IFS A of a non empty set X of the structure $A = \{x, \mu(x), \nu(x)\}$ Where $\mu(x): x \to [0,1]$ and $\nu(x): x \to [0,1]$ are membership and non membership functions ∋ for each $x \in X$ and we've $0 \leq \mu(x) + \nu(x) \leq 1$

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Definition 2.3[12] Let $X \neq \emptyset$. A MFS A in X is characterized by the set of ordered sequence as follows $A = \{(x, \mu_1(x), \mu_2(x), \mu_3(x) \ldots \ldots \mu_k(x) \ldots .) : x \in X\}$ Where $\mu_i(x) : x \to [0,1]\ \forall\, i$

Definition 2.4 [22] Let $X \neq \emptyset$. A IMFS A in X is characterized by the set

$$A = \{(x, (\mu_1(x), \mu_2(x), \mu_3(x) \ldots \ldots \mu_k(x) \ldots .), (\nu_1(x), \nu_2(x), \nu_3(x) \ldots \ldots \nu_k(x) . .) : x \in X\}$$

Where $\mu_i(x) : x \to [0,1]$, $\nu_i(x) : x \to [0,1]$ and we have $0 \leq \mu_i(x) + \nu_i(x) \leq 1\ \forall\, i = 1,2, \ldots .k$

Definition 2.5[22] Let $X \neq \emptyset$. A k-dimensional IMFS A in X is characterized by the set

$$A = \{(x, \mu_1(x), \mu_2(x), \mu_3(x) \ldots \ldots \mu_k(x)), (\nu_1(x), \nu_2(x), \nu_3(x) \ldots \ldots \nu_k(x)) : x \in X\}$$

Where $\mu_i(x) : x \to [0,1]$, $\nu_i(x) : x \to [0,1]$ and we have $0 \leq \mu_i(x) + \nu_i(x) \leq 1\ \forall\, i = 1,2, \ldots .k$

Definition 2.6[17] Let A be a IFS of a Ring R. Let $t \in [0,1]$ then $t -$ IFS of R with respect to IFS A and is characterized by $A^t = \left(\mu^t(x), \nu^t(x)\right)$ where $\mu^t(x) = \min\{\mu(x), t\}$ and $\nu^t(x) = \max\{\nu(x), 1 - t\}\ \forall\, x \in R$

Definition 2.7[23] Let A be a IMFS in x with dimension k. let $t \in [0,1]$ then the IMFS A^t of x is known as A^t-IMFS (t - IFMS) of x w.r.to IMFS A and if characterized by

$A^t = \{x, \mu(x), \nu(x) : x \in X\}$ where $\mu^t = \left(\mu^t{}_1(x), \mu^t{}_2(x), \mu^t{}_3(x) \ldots \ldots \mu^t{}_k(x)\right)$ and

$\nu^t = (\nu^t{}_1(x), \nu^t{}_2(x), \nu^t{}_3(x) \ldots \ldots \nu^t{}_k(x)) \ni 0 \leq \mu^t{}_i(x) + \nu^t{}_i(x) \leq 1 \forall\ x \in X$ and i=1,2,…k where

$\mu^t{}_i(x) = \min\{\mu^t{}_i(x), t\}$ and $\nu^t{}_i(x) = \max\{\nu^t{}_i(x), 1 - t\}$ and $\mu^t{}_1(x) \geq \mu^t{}_2(x) \geq \mu^t{}_3(x) \ldots \ldots \geq \mu^t{}_k(x)$ for all x in X

Definition 2.8 [17] An t IFS of R, let $t \in [0,1]$ then the A^t of R is known as $t -$ IFSR of R, if A^t is IFSR that is

(i) $\mu^t(x - y) \geq \min\{\mu^t(x), \mu^t(y)\}$ and $\nu^t(x - y) \leq \max\{\nu^t(x), \nu^t(y)\}$

(ii) $\mu^t(xy) \geq \min\left\{\mu^t(x), \mu^t(y)\right)$ and $\nu^t(xy) \leq \max\left\{\nu^t(x), \nu^t(y)\right)$ where $A^t = \left(\mu^t(x), \nu^t(x)\right)$

and $\mu^t(x) = \min\{\mu(x), t\}$, $\nu^t(x) = \max\{\nu(x), t\}$ for every $x, y, x^{-1} \in R$

Definition 2.9[23] An $t -$ IMFS $A^t = \left\{(x, \mu^t(x), \nu^t(x)) : x \in x\right\}$ of a Ring R is known as A^t – IMFSR of R if it fulfills the accompanying $\forall\, x, y \in R\ \&\ t \in [0,1]$

(i) $\mu^t(x - y) \geq \min\{\mu^t(x), \mu^t(y)\}$ and $\nu^t(x - y) \leq \max\{\nu^t(x), \nu^t(y)\}$

(ii) $\mu^t(xy) \geq \min\{\mu^t(x), \mu^t(y)\}$ and $\nu^t(xy) \leq \max\{\nu^t(x), \nu^t(y)\}$ where

$\mu^{t} = (\mu^{t}_1(xy), \mu^{t}_2(xy), \mu^{t}_3(xy) \ldots \mu^{t}_k(xy))$ and $\nu^{t} = (\nu^{t}_1(xy), \nu^{t}_2(xy), \nu^{t}_3(xy) \ldots \nu^{t}_k(xy))$ such that

$0 \le \mu^{t}_i(xy) + \nu^{t}_i(xy) \le 1$ for all $xy \in R$ and i=1,2,… k where

$\mu^{t}_i(xy) = \min\{\mu^{t}_i(xy), t\}$ and $\nu^{t}_i(x) = \max\{\nu^{t}_i(xy), 1-t\}$ and

$\mu^{t}_1(xy) \ge \mu^{t}_2(xy) \ge \mu^{t}_3(xy) \ldots \ge \mu^{t}_k(xy)$ for all $x, y \in R$

Definition 2.10[21] Let A be a IFS of a Ring R. Let $(t_1, t_2) \in [0,1]$ and $t_2 \le 1 - t_1$ then the IFS A' of are R is known as (t_1, t_2) − IFS of R with respect to IFS A and is characterized by $A' = \left(\mu'(x), \nu'(x)\right)$ where $\mu'(x) = \max\{\mu(x), t_1\}$ and $\nu'(x) = \max\{\nu(x), t_2\}$ for all $x \in R$

3.Properties of $(\alpha, \beta) - cuts$ of the (t_1, t_2) − Intuitionistic Multi Fuzzy Subring of a Ring

Definition 3.1 Let A be a IMFS in X with dimension k. let $(t_1, t_2) \in [0,1]$ then the IMFS A' of x is known as (t_1, t_2)-IMFS of X w.r.to IMFS A and if characterized by $A' = \{(x, \mu'(x), \nu'(x)) : x \in x\}$ where

$\mu'(x) = (\mu'_1(x), \mu'_2(x), \mu'_3(x) \ldots \mu'_k(x))$ and $\nu'(x) = (\nu'_1(x), \nu'_2(x), \nu'_3(x) \ldots \nu'_k(x))$

$\ni 0 \le \mu'_i(x) + \nu'_i(x) \le 1 \ \forall x \in x$ and i=1,2,…k where $\mu'_i(x) = \min\{\mu'_i(x), t_1\}$ and $\nu'_i(x) = \max\{\nu'_i(x), 1 - t_1\}$ and $\mu'_1(x) \ge \mu'_2(x) \ge \mu'_3(x) \ldots \ge \mu'_k(x) \ \forall x$ in X

Definition 3.2 Let $A' = \{(x, \mu'(x), \nu'(x)) : x \in X\}$ be an $(t_1, t_2) - IFMS$ and let $\alpha = (\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_k) \epsilon [0,1]^k$ and $\beta = (\beta_1, \beta_2, \beta_3 \ldots \beta_k) \epsilon [0,1]^k$ where each $\alpha_i, \beta_i \epsilon [0,1]$ with $0 \le \alpha_i + \beta_i \le 1 \forall i$ then the $(\alpha, \beta) - cut$ of A' is the set of all X suchthat $\mu'_i(x) \ge \alpha_i$ with the relating $\nu'_i(x) \le \beta_i \forall i$ and is meant by $[A']_{(\alpha,\beta)}$ obviously it's a crisp set

Definition 3.3 Let $A' = \{(x, \mu'(x), \nu'(x)) : x \in X\}$ be an $(t_1, t_2) - IFMS$ and let $\alpha = (\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_k) \epsilon [0,1]^k$ and $\beta = (\beta_1, \beta_2, \beta_3 \ldots \beta_k) \epsilon [0,1]^k$ where $\alpha_i, \beta_i \epsilon [0,1]$ with $0 \le \alpha_i + \beta_i \le 1 \forall i$ then the strong $(\alpha, \beta) - cut$ of A' is the set of all X suchthat $\mu'_i(x) > \alpha_i$ with the comparing $\nu'_i(x) < \beta_i \forall i$ and is indicated by $[A']^{*}_{(\alpha,\beta)}$ obviously it's likewise a crisp set

Definition 3.4 An $(t_1, t_2) - IFMS$ $A' = \{(x, \mu'(x), \nu'(x)) : x \in X\}$ of a Ring R is supposed to be a (t_1, t_2) − IMFSR of R ($(t_1, t_2) - IFMSR$) in the event that it fulfills the accompanying $\forall x, y \in R$ and $(t_1, t_2) \in [0,1]$

(i) $\mu'(x - y) \ge \min\{\mu'(x), \mu'(y)\}$ and $\nu'(x - y) \le \max\{\nu'(x), \nu'(y)\}$

(ii) $\mu'(xy) \ge \min\{\mu'(x), \mu'(y)\}$ and $\nu'(xy) \le \max\{\nu'(x), \nu'(y)\}$

where $ɥ' = (ɥ'_1(x y), ɥ'_2(x y), ɥ'_3(x y) \dots ..ɥ'_k(x y))$ and $ν' = (ν'_1(x y), ν'_2(x y), ν'_3(x y) \dots ..ν'_k(x y)) \ni$
$0 \leq ɥ'_i(x y) + ν'_i(x y) \leq 1 \; \forall \, x y \in R$ and i=1,2,… k

Theorem 3.5 Let A' and B' are any two $(t_1, t_2) - IFMS$ of dimension k taken from a nonempty set X then $A' \subseteq B'$ iff $[A']_{(\alpha,\beta)} \subseteq [B']_{(\alpha,\beta)}$ for every $\alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Example 3.6 Consider the Ring $\{z_5, +_5, \times_5\}$ where $Z_5 = \{0,1,2,3,4\}$ define IMFS A of dimension two Z_5 by

$$ɥ_{A_i}(x) = \left(ɥ_{A_1}(x), ɥ_{A_2}(x)\right) = \begin{cases} (0.6,0.7) if\ x = 0 \\ (0.5,0.3) if\ x = 1,3 \\ (0.4,0.3) if\ x = 2,4 \end{cases} \text{ and}$$

$$ν_{A_i}(x) = \left(ν_{A_1}(x), ν_{A_2}(x)\right) = \begin{cases} (0.2,0.1) if\ x = 0 \\ (0.6,0.4) if\ x = 1,2 \\ (0.7,0.5) if\ x = 3,4 \end{cases}$$

It is easy to verify that is A not IFMSR of Z_5, if we take $t_1 = 0.3$ then $ɥ_{A'_i}(x) = (0.3, 0.3 \text{ if } x = 0,1,2,3,4)$ and $t_2 = 0.7$ then $ν_{A'_i}(x) = (0.7, 0.7 \text{ if } x = 0,1,2,3,4)$ A' is IMFSR. let $(\alpha, \beta) - cut$ of A' and strong $(\alpha, \beta) - cut$ of A' ,
$\alpha_i, \beta_i \epsilon [0,1]$ if we take $\alpha_1 = 0.1$, $\alpha_2 = 0.3, \beta_1 = 0.7$ *and* $\beta_2 = 0.6$
$ɥ_{A'_i}(x) \geq \alpha_i$, $ν_{A'_i}(x) \leq \beta_i$ *and* $ɥ_{A'_i}(x) > \alpha_i$, $ν_{A'_i}(x) > \beta_i$ also satisfied

Definition 3.7 An $(t_1, t_2) - IMFSR$ of R , $A' = \left\{(x, ɥ'(x), ν'(x)) : x \in R\right\}$ of a Ring R is defined as $(t_1, t_2) - IMFNSR$ of R is satisfies (i) $ɥ'(x y) = ɥ'(y x)$ and (ii) $ν'(x y) = ν'(y x)$ for all $x, y \in R$ and $(t_1, t_2) \in [0,1]$

Definition 3.8 Let $(R, +, \cdot)$ be Ring A' , B' be any two $(t_1, t_2) - IMFS$ having a similar dimension k of R, then the multiple of A' & B' signified by $ɥ' \circ ν'$ is defined as

$ɥ' \circ ν'(x) = ɥ'_{u \circ v}(x), ν'_{u \circ v}(x) \forall x \in R$ where

$$ɥ'_{u \circ v}(x) = \begin{cases} \quad max\ [min\ \{ɥ'(y), ɥ'(z)\} : y z = x \forall\ y, z \in R] \\ \text{and } 0_k = (0,0,0, \dots 0_{k-times}) \text{ if } x \text{ is not expressible as } x = y z \end{cases}$$

$$ν'_{u \circ v}(x) = \begin{cases} \quad min\ [max\ \{ν'(y), ν'(z)\} : y z = x \forall\ y, z \in R] \\ \text{and } 0_k = (0,0,0, \dots 0_{k-times}) \text{ if } x \text{ is not expressible as } x = y z \end{cases}$$

$$ɥ' \circ ν'(x) = \begin{cases} max\ [min\ \{ɥ'(y), ɥ'(z)\} : y z = x \forall\ y, z \in R] \\ min\ [max\ \{ν'(y), ν'(z)\} : y z = x \forall\ y, z \in R] \\ \text{and } (0_k, 1_k) \text{if } x \text{ isn't expressible as } x = y z \end{cases} \quad \text{That is } x \in R$$

$$\text{ш}' \circ \vee'(x) = \begin{cases} max\ [min\ \{\text{ш}'(y), \text{ш}'(z)\} : yz = x \forall\ y, z \in R] \\ min\ [max\ \{\vee'(y), \vee'(z)\} : yz = x \forall\ y, z \in R] \\ \text{and } (0,1)_k \text{ if } x \text{ isn't expressible as } x = yz \end{cases} \quad \text{Where } (0,1)_k = ((0,1), (0,1), \ldots \ldots k \text{ times})$$

Definition 3.9 Let x and y be any two nonempty sets and $f: x \longrightarrow y$ *be a mapping. Let* A' *and* B' any two $(t_1,t_2) - IMFS$ of X and Y separately having a similar dimension k then the image(_img) of $A'(\subseteq x)$ under the map f is meant by $f(\text{ш}')$ as $\forall\ y \in Y$

$f(\text{ш}')(y) = (A'_{f(\text{ш}')}(y), B'_{f(\text{ш}')}(y))$ Where

$$A'_{f(\text{ш}')}(y) = \begin{cases} \max \{\text{ш}'(x) : x \in f^{-1}(y) \\ 0_k, \text{otherwise} \end{cases} \quad \text{and}$$

$$B'_{f(\text{ш})}(y) = \begin{cases} \min \{\vee'(x) : x \in f^{-1}(y) \\ 1_k, \text{otherwise} \end{cases} \quad \text{that is}$$

$$f(\text{ш}')(y) = \begin{cases} \left(\max \{A'_{\text{ш}_i}(x) : x \in f^{-1}(y), \min \{B'_{\text{ш}_i}(y) : x \in f^{-1}(y)\right)_{i=1}^{k} \\ \text{and } (0,1)_k, \text{ in any case where } (0,1)_k = (0,1), (0,1) \ldots \ldots k \text{ times} \end{cases}$$

Likewise pre_img of $B'(\subseteq y)$ under the map f is indicated by $f^{-1}(\vee')$ and it is characterized as $f^{-1}(\vee')(x)$ =($A'_{f(\text{ш}')}(y), B'_{f(\text{ш}')}(y)$), forevery $x \in X$

Proposition 3.10 If A' & B' are any two $(t_1,t_2) - \text{IMFS}$ of a universal set X then coming up next are hold

(i) $[A']_{(\alpha,\beta)} \subseteq [B']_{(\delta,\theta)}$ if $\alpha \geq \delta$ and $\beta \leq \theta$

(ii) $A' \subseteq B'$ Implies $[A']_{(\alpha,\beta)} \subseteq [B']_{(\delta,\theta)}$

(iii) $[A' \cup B']_{(\alpha,\beta)} = [A']_{(\alpha,\beta)} \cup [B']_{(\alpha,\beta)}$ (Here equality holds if $\alpha_i + \beta_i = 1 \forall i$)

(iv) $[A' \cap B']_{(\alpha,\beta)} = [A']_{(\alpha,\beta)} \cap [B']_{(\alpha,\beta)}$

(v) $[\cap A'_i]_{(\alpha,\beta)} = \cap [A'_i]_{(\alpha,\beta)}$ Where $\alpha, \beta \epsilon [0,1]^k$

Proposition 3.11 Let $(R, +, \cdot)$ be Ring A' , B' be any two $(t_1,t_2) - IMFSR$ of R and $\alpha, \beta \epsilon [0,1]^k$ then $[A']_{(\alpha,\beta)}$ is a Subring of R where $\text{ш}'_i(e) \geq \alpha$, $\vee'_i(e) \leq \beta$and 'e' is the identity component of R

Proof Since $\text{ш}'_i(e) \geq \alpha$ & $\vee'_i(e) \leq \beta$, $e \in [A']_{(\alpha,\beta)}$ therefore $[A']_{(\alpha,\beta)} \neq \emptyset$

Let $x, y \in [A']_{(\alpha,\beta)}$ then $\text{ш}'_i(x) \geq \alpha$, $\vee'_i(x) \leq \beta$ and $\text{ш}'_i(y) \geq \alpha$, $\vee'_i(y) \leq \beta$

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Then $\forall$ i $\mu'_i(x) \geq \alpha_i$, $\mu'_i(x) \leq \beta_i$ and $\nu'_i(y) \geq \alpha_i$, $\nu'_i(y) \leq \beta_i$

$\Rightarrow min\{\mu'_i(x), \mu'_i(y)\} \geq \alpha_i$ and $max\left\{\nu'_i(x), \nu'_i(y)\right\} \leq \beta_i \; \forall i$ $-----------(1)$

$\Rightarrow \mu'_i(x - y) \geq \min\{\mu'_i(x), \mu'_i(y)\} \geq \alpha_i$ and $\nu'_i(x - y) \leq \max\{\nu'_i(x), \nu'_i(y)\} \leq \beta_i \; \forall i$

And we have $\mu'_i(xy) \geq \min\{\mu'_i(x), \mu'_i(y)\} \geq \alpha_i$ and $\nu'_i(xy) \leq \max\{\nu'_i(x), \nu'_i(y)\} \leq \beta_i \; \forall i$ Since A' is an $(t_1,t_2) - IMFSR$ of a Ring R and by (1)

$\Rightarrow \mu'_i(x - y) \geq \alpha_i$ and $\nu'_i(x - y) \leq \beta_i \forall i \Rightarrow x - y \in [A']_{(\alpha,\beta)}$ and

$\Rightarrow \mu'_i(x\, y) \geq \alpha_i$ and $\nu'_i(x\, y) \leq \beta_i \forall i$

$\Rightarrow x\, y \in [A']_{(\alpha,\beta)}$

$\Rightarrow [A']_{(\alpha,\beta)}$ is a subring of R

Theorem 3.12 If A' is a $(t_1,t_2) - IMFS$ of a Ring R, then A' is an $(t_1,t_2) - IMFSR$ of R $\Leftrightarrow$ each $[A']_{(\alpha,\beta)}$ is a subring of R for all $\alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Proof $(\Rightarrow)$ Let A' be a $(t_1,t_2) - IMFSR$ of Ring R then each $[A']_{(\alpha,\beta)}$ is a subring of R for all $\alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

$(\Leftarrow)$ Conversely, let A' be a $(t_1,t_2) - IMFS$ of R we must prove

(i) $\mu'(x - y) \geq \min\{\mu'(x), \mu'(y)\}$ and $\nu'(x - y) \leq \max\{\nu'(x), \nu'(y)\}$

(ii) $\mu'(xy) \geq \min\{\mu'(x), \mu'(y)\}$ and $\nu'(xy) \leq \max\{\nu'(x), \nu'(y)\}$ for all $x, y \in R$

Let $x, y \in R$ and $\forall i$, let $\alpha_i = \min\{\mu'_i(x), \mu'_i(y)\}$ and $\beta_i = \max\{\nu'_i(x), \nu'_i(y)\}$ then $\forall i$

We have $\mu'_i(x) \geq \alpha_i, \mu'_i(y) \geq \alpha_i$ and $\nu'_i(x) \leq \beta_i, \; \nu'_i(y) \leq \beta_i$ that is $\forall i$

we have $\mu'_i(x) \geq \alpha_i, \; \nu'_i(x) \leq \beta_i$ and, $\mu'_i(y) \geq \alpha_i, \; \nu'_i(y) \leq \beta_i$ then $\mu'(x) \geq \alpha, \nu'(x) \leq \beta$ and $\mu'(y) \geq \alpha, \nu'(y) \leq \beta$ ie., $x \in [A']_{(\alpha,\beta)}$ and $y \in [A']_{(\alpha,\beta)}$ therefore $x - y, x\, y \in [A']_{(\alpha,\beta)}$ since each $[A']_{(\alpha,\beta)}$ is a Subring by hypothesis, therefore $\forall i$ we have $\mu'_i(x - y) \geq \alpha_i = \min\{\mu'_i(x), \mu'_i(y)\}$ and $\nu'_i(x - y) \leq \beta_i = \max\{\nu'_i(x), \nu'_i(y)\} \forall i$ ie., $\mu'(x - y) \geq \min\{\mu'(x), \mu'(y)\}$ and $\nu'(x - y) \leq \max\{\nu'(x), \nu'(y)\}$

hence (i) is true

Now let $\mathfrak{x} \in R$ and $\forall i$, let $\mu'_i(\mathfrak{x}) = \alpha_i$ and $\nu'_i(\mathfrak{x}) = \beta_i$ then $\mu'_i(\mathfrak{x}) \geq \alpha_i$ *and* $\nu'_i(\mathfrak{x}) \leq \beta_i$ is true $\forall i$ then $\mu'(\mathfrak{x}) \geq \alpha$ and $\nu'(\mathfrak{x}) \leq \beta$ thus $\mathfrak{x} \in [A']_{(\alpha,\beta)}$ also we have $\mu'_i(\mathfrak{x}\mathfrak{y}) \geq \alpha_i = \min\{\mu'_i(\mathfrak{x}), \mu'_i(\mathfrak{y})\}$ and $\nu'_i(\mathfrak{x}\mathfrak{y}) \leq \beta_i = \max\{\nu'_i(\mathfrak{x}), \nu'_i(\mathfrak{y})\} \forall i$ ie., $\mu'(\mathfrak{x}\mathfrak{y}) \geq \min\{\mu'(\mathfrak{x}), \mu'(\mathfrak{y})\}$ and $\nu'(\mathfrak{x}\mathfrak{y}) \leq \max\{\nu'(\mathfrak{x}), \nu'(\mathfrak{y})\}$

hence (ii) is true.

Now let $\mathfrak{x} \in R$ & $\forall i$ let let $\mu'_i(\mathfrak{x}) = \alpha_i$ & $\nu'_i(\mathfrak{x}) = \beta_i$ then $\mu'_i(\mathfrak{x}) \geq \alpha_i$ & $\nu'_i(\mathfrak{x}) \leq \beta_i$ is true $\forall i$ then $\mu'(\mathfrak{x}) \geq \alpha$ & $\nu'(\mathfrak{x}) \leq \beta$ thus $\mathfrak{x} \in [A']_{(\alpha,\beta)}$ hence A' is an $(t_1,t_2) - IMFSR$ of R

Theorem 3.13 If A' and B' be any two $(t_1,t_2) - IMFSR$ of a Ring R, then $[A' \cap B']$ is also an $(t_1,t_2) - IMFSR$ of R

Proof By above theorem A' is an $(t_1,t_2) - IMFSR$ of a Ring R $\Leftrightarrow$ each $[A']_{(\alpha,\beta)}$ is a subring of R for all $\alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$ But since $[A' \cap B']_{(\alpha,\beta)} = [A']_{(\alpha,\beta)} \cap [B']_{(\alpha,\beta)}$ and both $[A']_{(\alpha,\beta)}$ and $[B']_{(\alpha,\beta)}$ are subring of R(as A' and B' are $(t_1,t_2) - IMFSR$) and the intersection of any two subrings is also a subring of R which implies that $[A' \cap B']_{(\alpha,\beta)}$ is a subring of R and hence $[A' \cap B']$ is an $(t_1,t_2) - IMFSR$ of R

Remark 3.14 The union of $(t_1,t_2) - IMFSR$ of a Ring R need not be an $(t_1,t_2) - IMFSR$ of the Ring R

Theorem 3.15 If A' and B' be any two $(t_1,t_2) - IMFSR$ of a Ring R, then $A' \circ B'$ is a $(t_1,t_2) - IMFSR$ of R $\Leftrightarrow A' \circ B' = B' \circ A'$

Proof Since A' and B' be any two $(t_1,t_2) - IMFSR$ of R each $[A']_{(\alpha,\beta)}$ and $[B']_{(\alpha,\beta)}$ are subring of R for all $\alpha, \beta \epsilon [0,1]^k$ With $0 \leq \alpha_i + \beta_i \leq 1 \forall i$-----------(1)

Suppose $A' \circ B'$ is a $(t_1,t_2) - IMFSR$ of R $\Leftrightarrow$ each $[A' \circ B']_{(\alpha,\beta)}$ are subring of R $\forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$. Now from (1) $[A']_{(\alpha,\beta)} \circ [B']_{(\alpha,\beta)}$ is a subring of R

$\Leftrightarrow [A']_{(\alpha,\beta)} \circ [B']_{(\alpha,\beta)} = [B']_{(\alpha,\beta)} \circ [A']_{(\alpha,\beta)}$

if H and K are any two subring then HK is a subring of R $\Leftrightarrow$HK = KH

$\Leftrightarrow [A' \circ B']_{(\alpha,\beta)} = [B' \circ A']_{(\alpha,\beta)} \forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i \Leftrightarrow A' \circ B' = B' \circ A'$

Theorem 3.16 If A' is any $(t_1,t_2) - IMFSR$ of a Ring R then $A'.A' = A'$

Proof Since A' *is a* $(t_1,t_2) - IMFSR$ of a Ring R each $[A']_{(\alpha,\beta)}$ is a subring of R $\forall \alpha, \beta \epsilon [0,1]^k$

with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

$\Rightarrow [A']_{(\alpha,\beta)} . [A']_{(\alpha,\beta)} = [A']_{(\alpha,\beta)}$ since H is a subring of R $\Rightarrow$ H H = H

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$\Rightarrow [A'.A'']_{(\alpha,\beta)}=[A']_{(\alpha,\beta)}\ \forall\alpha,\beta\epsilon[0,1]^k$ with $0\leq\alpha_i+\beta_i\leq 1\forall i$

$\Rightarrow A'.A'=A'$

4. $(t_1,t_2)-$ Intuitionistic Multi Fuzzy Cosets of a Ring

Definition 4.1 Let R be Ring and A' be a $(t_1,t_2)-IMFSR$ of a Ring R. Let $x\epsilon R$ be a fixed element then the set $xA'=\{\gamma,\mu_{xA'}(x),\nu_{xA'}(y):\gamma\epsilon R\}$ Where $\mu_{xA'}(\gamma)=\mu_{A'}(x^{-1}\gamma)$ and $\nu_{xA'}(\gamma)=\nu_{A'}(x^{-1}\gamma)\ \forall\gamma\epsilon R$ is known as the $(t_1,t_2)-Intuitionistic\ multi\ fuzzy\ coset\ of\ R$ characterized by A' and x likewise the set $A'x=\{\gamma,\mu_{A'x}(x),:\gamma\epsilon R\}$ where $\mu_{xA'}(\gamma)=\mu_{A'}(\gamma x^{-1})$ and $\nu_{xA'}(\gamma)=\nu_{A'}(\gamma x^{-1})\ \forall\gamma\epsilon R$ is known the $(t_1,t_2)-Intuitionistic\ multi\ fuzzy\ coset\ of\ R$ characterized by A' and x

Remark 4.2 It clear that if A' is a $(t_1,t_2)-$ IMFNSR of R, then the $(t_1,t_2)-$ IMF right coset and $(t_1,t_2)-$ IMF left coset of A' on R matches and this case, we just call it as $(t_1,t_2)-$ IMFCS(intuitionistic multi fuzzy coset)

Example 4.3 Let R be a ring then $A'=\{\mu_{A'}(x),\nu_{A'}(x):x\epsilon R|\mu_{A'}(x)=\mu_{A'}(e)\text{ and }\nu_{A'}(x)=\nu_{A'}(e)\}$ is a $(t_1,t_2)-$ IMFNSR of R

Theorem 4.4 Let A' be a $(t_1,t_2)-IMFSR$ of a Ring R. furthermore, x be any proper component of R then, at that point, the accompanying hold

(i) $x\,[A']_{(\alpha,\beta)}=[xA']_{(\alpha,\beta)}$

(ii) $[A']_{(\alpha,\beta)}x=[A'x]_{(\alpha,\beta)}\ \forall\alpha,\beta\epsilon[0,1]^k$ with $0\leq\alpha_i+\beta_i\leq 1\forall i$

Proof (i) $x\,[A']_{(\alpha,\beta)}=\{\gamma\epsilon R:\mu_{A'x}(\gamma)\geq\alpha$ and $\nu_{A'x}(\gamma)\leq\beta$ with $0\leq\alpha_i+\beta_i\leq 1\forall i$

Also $x\,[A']_{(\alpha,\beta)}=x\left\{\{y\epsilon R:\mu_{A'}(y)\geq\alpha\text{ and }\nu_{A'}(y)\leq\beta\right\}$--------(1)

$=\{xy\epsilon R;\mu_{A'}(y)\geq\alpha\text{ and }\nu_{A'}(y)\leq\beta\}$

Put $xy=\gamma\Rightarrow y=(x^{-1}\gamma)$ then (1) can be written as

$x\,[A']_{(\alpha,\beta)}=\{xy\epsilon R;\mu_{A'}(y)\geq\alpha\text{ and }\nu_{A'}(y)\leq\beta\}$

$=\{xy\epsilon R;\mu_{xA'}(xy)\geq\alpha\text{ and }\nu_{xA'}(xy)\leq\beta\}$

$=[xA']_{(\alpha,\beta)}$

Therefore, $x\,[A']_{(\alpha,\beta)}=[xA']_{(\alpha,\beta)}\ \forall\alpha,\beta\epsilon[0,1]^k$ with $0\leq\alpha_i+\beta_i\leq 1\forall i$

(ii) Now $[A'x]_{(\alpha,\beta)} = \{xy \in R ; \mu_{xA'}(xy) \geq \alpha \text{ and } \nu_{xA'}(xy) \leq \beta\}$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Also $[A']_{(\alpha,\beta)}x = \{y \in R ; \mu_{A'}(y) \geq \alpha \text{ and } \nu_{A'}(y) \leq \beta\}$

$= \{yx \in R ; \mu_{A'}(y) \geq \alpha \text{ and } \nu_{xA'}(y) \leq \beta\}$------------(2)

Also (2) can be written as ,

$[A']_{(\alpha,\beta)}x = \{xy \in R ; \mu_{A'}(y) \geq \alpha \text{ and } \nu_{A'}(y) \leq \beta\}$

$= \{xy \in R ; \mu_{A'_x}(xy) \geq \alpha \text{ and } \nu_{A'_x}(xy) \leq \beta\}$

$= [A'x]_{(\alpha,\beta)}$

Therefore, $[A']_{(\alpha,\beta)}x = [A'x]_{(\alpha,\beta)}$ $\forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Theorem 4.5 Let A' be a $(t_1, t_2) - IMFSR$ of a Ring R.let x, y be any two elements of R suchthat $\alpha = min\{\mu_{A'}(x), \mu_{A'}(y)\}$ & $\beta = max\{\nu_{A'}(x), \nu_{A'}(y)\}$ then the accompanying hold

(i) $xA' = yA' \Leftrightarrow xy \epsilon [A']_{(\alpha,\beta)}$

(ii) $A'x = A'y \Leftrightarrow yx \epsilon [A']_{(\alpha,\beta)}$

Proof (i) If $xA' = yA' \Leftrightarrow [xA']_{(\alpha,\beta)} = [yA']_{(\alpha,\beta)}$ $\forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

$\Leftrightarrow x[A']_{(\alpha,\beta)} = y[A']_{(\alpha,\beta)}$

$\Leftrightarrow xy \epsilon [A']_{(\alpha,\beta)}$ Since each $[A']_{(\alpha,\beta)}$ is a subring of R

(ii) If $A'x = A'y \Leftrightarrow [A'x]_{(\alpha,\beta)} = [A'y]_{(\alpha,\beta)}$ $\forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

$\Leftrightarrow [A']_{(\alpha,\beta)}x = [A']_{(\alpha,\beta)}y$

$\Leftrightarrow xy \epsilon [A']_{(\alpha,\beta)}$ Since each $[A']_{(\alpha,\beta)}$ is a subring of R

5.Homomorphism of $(t_1, t_2) -$ Intuitionistic Multi Fuzzy subring

Proposition 5.1 Let $f: x \rightarrow y$ be an onto map if A' and B' are $(t_1, t_2) - IMFS$ with dimensions k of x & y respectively hold

(i) $f[A']_{(\alpha,\beta)} \subseteq [f(A')]_{(\alpha,\beta)}$

(ii) $f^{-1}([B']_{(\alpha,\beta)}) = [f^{-1}(B')_{(\alpha,\beta)}]$ $\forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Theorem 5.2 Let $f: R_1 \longrightarrow R_2$ be an onto _homo and if A' is an $(t_1, t_2) - IMFS$R of R_1 then $f(A')$ is an $(t_1, t_2) - IMFS$ R of R_2

Proof By theorem demonstrating that each is sufficient $[f(A')]_{(\alpha,\beta)}$ is a Subring of $R_2 \forall \alpha, \beta \epsilon [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1 \forall i$

Let $y_1, y_2 \epsilon [f(A')]_{(\alpha,\beta)}$ then,

$\mu_{f(A')}(y_1) \geq \alpha$, $\nu_{f(A')}(y_1) \leq \beta$ and $\mu_{f(A')}(y_2) \geq \alpha$, $\nu_{f(A')}(y_2) \leq \beta$

$\mu_{f(A'_i)}(y_1) \geq \alpha_i$, $\nu_{f(A'_i)}(y_1) \leq \beta_i$ and $\mu_{f(A'_i)}(y_2) \geq \alpha_i$, $\nu_{f(A'_i)}(y_2) \leq \beta_i$ -------------(1)

By proposition we have $f[A']_{(\alpha,\beta)} \subseteq [f(A')]_{(\alpha,\beta)}$ $\forall A' \epsilon (t_1, t_2) - IMFS$R of R_1 , since f is onto $\exists$ & x_2 in R_1

suchthat $f(x_1) = y_1, f(x_2) = y_2$ therefore (1) can be written

as $\mu_{f(A'_i)} f(x_1) \geq \alpha_i$, $\nu_{f(A'_i)} f(x_1) \leq \beta_i$ & $\mu_{f(A'_i)} f(x_2) \geq \alpha_i$, $\nu_{f(A'_i)} f(x_2) \leq \beta_i \forall i$

$\Rightarrow \mu_{A'_i} f(x_1) \geq \mu_{f(A'_i)} f(x_1) \geq \alpha_i$, $\nu_{A'_i} f(x_1) \leq \nu_{f(A'_i)} f(x_1) \leq \beta_i$ &

$\mu_{A'_i} f(x_2) \geq \mu_{f(A'_i)} f(x_2) \geq \alpha_i$, $\nu_{A'_i} f(x_2) \leq \nu_{f(A'_i)} f(x_2) \leq \beta_i \forall i$

$\Rightarrow \mu_{A'_i} f(x_1) \geq \alpha_i$, $\nu_{A'_i} f(x_1) \leq \beta_i$ and $\mu_{A'_i} f(x_2) \geq \alpha_i$, $\nu_{A'_i} f(x_2) \leq \beta_i \forall i$

$\Rightarrow \mu_{A'}(x_1) \geq \alpha$, $\nu_{A'}(x_1) \leq \beta$ and $\mu_{A'}(x_2) \geq \alpha$, $\nu_{A'}(x_2) \leq \beta$

$\Rightarrow min\{\mu_{A'}(x_1), \mu_{A'}(x_2)\} \geq \alpha$ and $max\{\nu_{A'}(x_1), \nu_{A'}(x_2)\} \leq \beta$

$\Rightarrow \mu_{A'}(x_1, x_2) \geq min\{\mu_{A'}(x_1), \mu_{A'}(x_2)\} \geq \alpha$ and

$\nu_{A'}(x_1, x_2) \leq max\{\nu_{A'}(x_1), \nu_{A'}(x_2)\} \leq \beta$

Since $A' \epsilon (t_1, t_2) - IMFS$R of R_1

$\Rightarrow \mu_{A'}(x_1, x_2) \geq \alpha$ and $\nu_{A'}(x_1, x_2) \leq \beta$

$\Rightarrow x_1, x_2 \epsilon [A']_{(\alpha,\beta)} \Rightarrow f(x_1, x_2) \epsilon [f(A')]_{(\alpha,\beta)} \Rightarrow f(x_1) f(x_2) \epsilon [f(A')]_{(\alpha,\beta)}$

$\Rightarrow y_1, y_2 \epsilon [A']_{(\alpha,\beta)} \Rightarrow [f(A')]_{(\alpha,\beta)}$ is a subring of R_2 $\forall \alpha, \beta \epsilon [0,1]^k$

$\Rightarrow$ $f(A')\epsilon(t_1,t_2) - IMFS$R of R_2

Corollary 5.3 If $f: R_1 \longrightarrow R_2$ be a _homo of a ring R_1 onto a ring R_2 and $\{A'_i : i \in I\}$ be a group of $(t_1,t_2) - IMFS$R of R_1 then $f(\cap A'_i)$ is a $(t_1,t_2) - IMFS$R of R_2

Theorem 5.4 If $f: R_1 \longrightarrow R_2$ be _homo of a ring R_1 into a ring R_2. If B' is an $(t_1,t_2) - IMFS$R of R_2 then $f^{-1}(B')$ is also a $(t_1,t_2) - IMFS$R of R_1

Proof By hypothesis, it is sufficient to demonstrate that $[f^{-1}(B')]_{(\alpha,\beta)}$ is a subring of R_1 with $0 \le \alpha_i + \beta_i \le 1 \forall i$ let $x_1, x_2 \ge \alpha$, $u_{f^{-1}(B')}(x_1) \ge \alpha$, $v_{f^{-1}(B')}(x_1) \le \beta$ and

$u_{f^{-1}(B')}(x_2) \ge \alpha$, $v_{f^{-1}(B')}(x_2) \le \beta$

$\Rightarrow$ $u_{(B')}(f(x_2)) \ge \alpha, v_{(B')}(f(x_2)) \le \beta$

$\Rightarrow min\{u_{B'}f(x_1), u_{B'}f(x_2)\} \ge \alpha$ & $max\{v_{B'}f(x_1), v_{B'}f(x_2)\} \le \beta$

$\Rightarrow u_{B'}(f(x_1)\, f(x_2)) \ge min\{u_{B'}f(x_1), u_{B'}f(x_2)\} \ge \alpha$ &

$v_{B'}(f(x_1)\, f(x_2)) \le max\{u_{B'}f(x_1), u_{B'}f(x_2)\} \le \beta$

Since $B'\epsilon(t_1,t_2) - IMFS$R of R

$\Rightarrow$ $(f(x_1)\, f(x_2))\epsilon[B']_{(\alpha,\beta)} \Rightarrow$ $f(x_1x_2)\epsilon[B']_{(\alpha,\beta)}$ since f is_homo

$\Rightarrow$ $x_1x_2 \,\epsilon\, f^{-1}[[B']_{(\alpha,\beta)}] = [f^{-1}[B']_{(\alpha,\beta)}]$

$\Rightarrow x_1x_2 \,\epsilon\, [f^{-1}[B']_{(\alpha,\beta)}] \Rightarrow$ $f^{-1}[B']_{(\alpha,\beta)}$ is a subring R_1

$\Rightarrow$ $f^{-1}[B']_{(\alpha,\beta)}$ *is a* $(t_1, t_2) - IMFS$R of R_1

Theorem 5.5 If $f: R_1 \longrightarrow R_2$ be a Surjective ring _homo and if A' is a $(t_1,t_2) - IMFNS$R of a ring R_1 then $f(A')$ is also $(t_1,t_2) - IMFNS$R of a ring R_2

Proof Since A' is a $(t_1,t_2) - IMFNS$R of a ring R_1 , let $y_1, y_2 \,\epsilon\, R_2$ be any element then there exist

same $x_1, x_2 \,\epsilon\, R_1$ suchthat $f(x_1) = y_1$ & $f(x_2) = y_2$.

Now $(f(A'))(y_1y_2)$ = ($u_{A'}(y_1y_2)$, $v_{A'}(y_1y_2)$) then prove that

$u_{f(A')}(y_1y_2) = u_{f(A')}(y_2y_1)$ and $v_{f(A')}(y_1y_2) = v_{f(A')}(y_2y_1)$

$\mu_{f(A'_i)}(y_1 y_2) = \mu_{f(A'_i)}(f(x_1)f(x_2)) = \mu_{f(A'_i)}(f(x_1 x_2)) = \mu_{A'_i}(x_1 x_2) \geq \min\{\mu_{A'}(x_1), \mu_{A'}(x_2)\}\ \forall x_1, x_2 \in R_1$ Such that $f(x_1) = y_1$ and $f(x_2) = y_2$ hence $\mu_{f(A'_i)}(y_1 - y_2)$ is an upperbound for all $\min\{\mu_{A'_i}(x_1), \mu_{A'_i}(x_2)\}\ \forall x_1, x_2 \in R_1$ since $\max\{\min\{\mu_{A'_i}(x_1), \mu_{A'_i}(x_2)\}\}$ is least upper bound $\mu_{A'}(x_1 x_2) \geq \max\{\min\{\mu_{A'_i}(x_1), \mu_{A'_i}(x_2)\}\}$

$= \min\{\max\{\mu_{A'_i}(x_1) : f(x_1) = y_1\}, \max\{\mu_{A'_i}(x_2) : f(x_2) = y_2\}\}$

$= \min\{\mu_{f(A'_i)}(y_1), \mu_{f(A'_i)}(y_2)\}$

$\mu_{f(A'_i)}(x_1 x_2) \geq \min\{\mu_{f(A'_i)}(y_1), \mu_{f(A'_i)}(y_2)\}$

$\mu_{f(A'_i)} f(x_2 x_1) = \mu_{f(A'_i)}(f(x_2)f(x_1)) = \mu_{f(A'_i)}(y_2 y_1)$

Likewise we can prove $\nu_{f(A'_i)}(y_1 y_2) = \nu_{f(A'_i)}(y_2 y_1)$

Hence the theorem

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